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A NUMERICAL OPTIMIZATION APPROACH TO TUNE A FIXED-STRUCTURE CONTROLLER FOR DAMPING CONTROL OF FEED DRIVE SYSTEMS

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ABSTRACT

Due to recent technological trends in high-speed, high-acceleration NC machine tools, the vibration control for a feed drive system in NC machine tools is regarded of more importance. This paper presents a tuning methodology of a fixed-structure feedback controller for the damping control of feed drive systems. Unlike celebrated optimal control theories, the present tuning methodology is based on a local search algorithm, and thus cannot always guarantee to find the globally optimal solution. However, it offers much more flexibility on the setup of the tuning objective and constraints, which is crucial for practical controller design. As an application example, the tuning of a fifth-order linear feedback controller for a feed drive system in an NC machine tool is presented. Unlike a notch filter, which is often used in conventional CNC units to cancel the mechanical resonance, the designed controller does not introduce much phase lag into the feedback loop, and thus offers a better closed-loop control performance.

Key words: controller tuning, damping control, numerical optimization, feed drive systems, frequency-domain loop-shaping design.

1 Introduction

Recent technological development has commercialized high-speed, high-acceleration machining centers of the feedrate up to 60 m/min, and the acceleration rate up to 1G. In feed drive systems driven by a servo motor and a ball screw, the torque is transmitted to a table via a coupling and a ball screw. Therefore, the dynamics of feed drive system always includes internal vibration modes due to the linear and torsional vibration of the ball screw. Such vibration is a more critical issue in high-speed feed drives (e.g. [1]). High-speed feed drives typically adopt a roller guideway, which generally exhibits lower damping compared to slide guideways. Furthermore, a ball screw of higher lead also causes lower damping. Other critical vibration issues include the structural vibration of a machine. Higher acceleration often introduces a low-frequency structural vibration to a machine base.

A typical servo controller in commercial CNCs uses a P (Proportional) controller in the position feedback loop and a PI (Proportional and Integral) controller in the velocity feedback loop. Although some latest CNCs additionally adopt a higher order filter to compensate specific problems, in most cases it is a filter of a simple structure such as the notch filter (see Section 2.1), which can be manually tuned by an engineer. Optimal control theories (e.g. [2]), which have been a subject of intensive research for decades in academia, can be used to design a high-order controller (filter) such that it exhibits "optimal" control performance. It is, however, widely recognized that there have not been many practical applications of such theories. One of critical issues is that such theories explicitly and implicitly impose many restrictions, which are difficult to be satisfied in practical applications. For example, as is well known, the order of H_2- and H_∞-optimal controllers must be the same as that of the plant model, and thus it often becomes too high for practical implementation.

This paper presents a design methodology of a fixed-structure feedback controller based on the frequency loop-shaping technique. Design requirements are given in the form of the "desired shape" of the system's frequency response plot, just as in the H_∞ loop-shaping design. However, unlike the H_∞ controller design, the order and structure of controller can be specified arbitrarily, and any additional requirements or constraints can be imposed. The problem generally becomes a nonconvex optimization problem. Since it is practically impossible to globally solve the problem in an efficient manner, we employ a local search algorithm. In many practical problems, local search algorithms show satisfactory search performance, and it at least "improves" the control performance. The present method offers an intuitive and efficient way to incorporate the designer's expertise and understanding of the physical system and design objectives into the controller design.

2 Tuning of a Fixed-structure Controller by Frequency-domain Loop-shaping

2.1 Problem Statement

This section illustrates the proposed controller tuning method by presenting an application example. Figure 1 depicts the schematics of a feed drive system driven by a rotary servo motor and a ball screw.

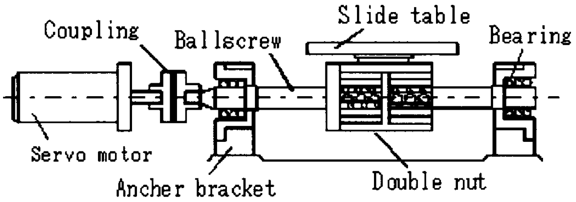

Figure 1. Schematic view of feed drive system

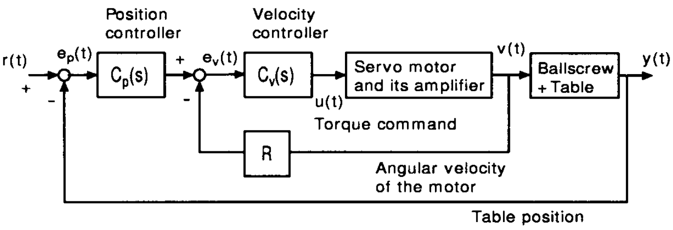

Figure 2. Overview of a typical control system for a feed drive system

Table 1. Major specifications of feed drive test stand

Servo motor	Rated output, kW	0.2
	Rated/Maximum torque, Nm	0.64 / 1.9
	Rated/Maximum rotation speed, rpm	3,000 / 4,000
	Moment of inertia, kg·cm^2	0.35
	Resolution of rotary encoder, pulse/rev	8,192
Ball screw	Diameter, m	0.2
	Lead, m	0.02
Linear scale	Resolution, μm/pulse	0.2

Figure 2 shows an overview of the servo control system. This structure is common in most CNC units commercially available in today's market.

Throughout this application example, a test stand of one-axis feed drive system is used in experimentation. Its major specifications are shown in Table 1. Figure 3 (solid line) shows the dynamics of the servo motor used in the test stand. The frequency response from the torque command to the motor's angular velocity (the transfer function from $u(s)$ to $v(t)$ in Figure 2) was measured by using the frequency sweep. There is a resonance at about 250 rad/sec. If the velocity controller, $C_v(s)$, is a PI controller just as in typical commercial CNCs, it can be easily seen that the resonance cannot be reduced without sacrificing the bandwidth, no matter how PI gains are tuned. The objective of this application example is to design $C_v(s)$ such that better damping performance can be obtained without sacrificing the bandwidth.

We note that it is a common practice for servo engineers to implement a compensation filter to reduce the mechanical resonance mode. For example, in many commercial CNC units, a notch filter of the following transfer function is often used to cancel the resonance:

$$G_{notch}(z) = \frac{1+z^{-n}}{2} \tag{1}$$

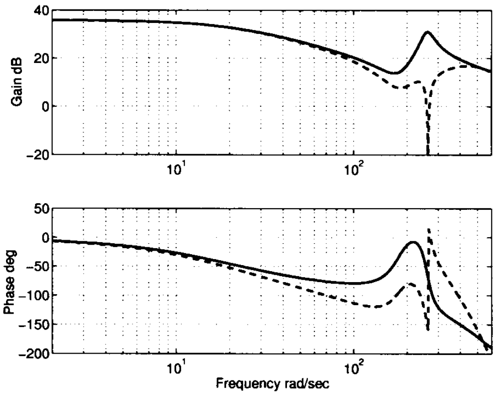

Figure 3. Frequency response of servo motor dynamics (solid: servo motor only, dashed: with the notch filter)

where n ($n = 1, 2, \cdots$) is a design parameter that determines the notch frequency. In Figure 3 (dashed line), the combined frequency response of the servo motor and a notch filter is also shown. Although the resonance is eliminated, significant phase lag is also introduced at frequencies lower than the notch frequency. Therefore, when the resonance frequency is relatively low and is close to the crossover frequency, the notch filter should not be used, since it prevents to raise the position loop gain and thus narrows the bandwidth of the closed-loop system. This is a well-known disadvantage of the notch filter.

2.2 Frequency-domain Loop-shaping Design as a Nonconvex Optimization Problem

Figure 4 illustrates inputs and outputs of the controller tuning. In this example, the designer is required to specify the following design requirements and constraints:

The order and structure of controller: The order and structure of controller can be specified arbitrarily. In this example, suppose that the controller structure is given as follows:

$$C(s) = \frac{g \prod_{i=1}^{\beta_1} (s+d_i) \prod_{i=1}^{\beta_2} (s^2+e_i s+f_i)}{\prod_{i=1}^{\alpha_1} (s+a_i) \prod_{i=1}^{\alpha_2} (s^2+b_i s+c_i)} \tag{2}$$

where $K := (g, a_1, b_1, b_2, c_1, c_2, d_1, e_1, e_2, f_1, f_2) \in \mathbb{R}^{11}$ is a set of tunable parameters. Let $\alpha_1 = 1$ (the number of real poles), $\alpha_2 = 2$ (the number of possibly complex poles), $\beta_1 = 1$ (the number of real zeros), and $\beta_2 = 2$ (the number of possibly complex poles).

"Desired shape" of gain plot: The designer is assumed to be capable to specify design requirements in the frequency domain, just as in the H_∞ loop-shaping design approach. Figure 5 (solid line) shows the measured frequency response of the velocity open-loop system (the transfer function from $e_v(s)$ to $v(s)$ in Figure 2) with a PI velocity-loop controller ($C_v(s) = K_{vp}(1 + K_{vi}/s)$ with $K_{vp} = 0.05$ and $K_{vi} = 800$). The original PI controller was manually tuned such that the best performance is obtained.

Plus marks in Figure 5 represent the "desired shape" of the open-loop frequency response, manually given by a designer. They are chosen based on design requirements to cancel the mechanical resonance

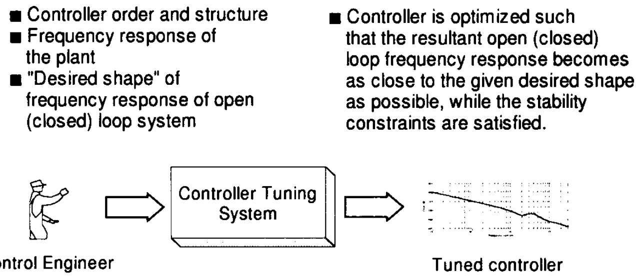

Figure 4. Controller tuning

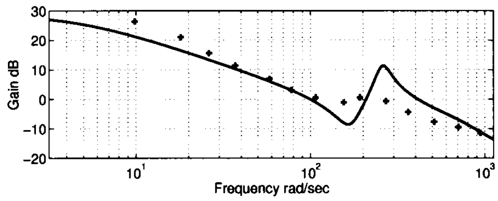

Figure 5. Measured velocity open-loop gain plot (solid line) and its "desired shape" (plus marks)

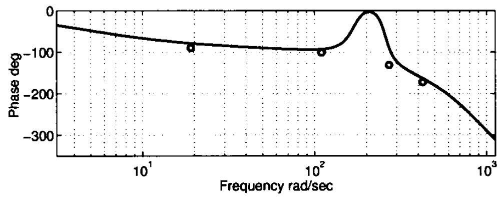

Figure 6. Measured velocity open-loop phase plot (solid line) and phase constraints given by designer (circles)

and counter resonance, without sacrificing the bandwidth. They are interpolated by using a spline curve and re-sampled at frequencies ω_k $(k = 1, \cdots, N_{gain})$. The desired gain shape is denoted by $W_{desired}(\omega_k)$ $(k = 1, \cdots, N_{gain})$.

Note that the choice of the transfer function on which design requirements are imposed is arbitrary, unlike the H_∞ loop-shaping design. That is, the "desired shape" does not have to be given on the open-loop transfer function; for example, it can be given on the sensitivity transfer function or the complementary sensitivity transfer function, depending on given design requirements.

Phase constraints: Note that no mathematical model of the plant dynamics is used in the design procedure proposed in this paper. As a heuristic way to secure the stability of the closed-loop system, phase constraints are imposed to the open-loop system. In Figure 6, the circles represent phase constraints given by the designer. The controller will be designed such that the phase plot of the open-loop system stays "above" the circles. Although the choice of phase constraints is not trivial, it should not be difficult for an experienced engineer. In this example, phase constraints are designed based on the phase plot with the original controller.

Optimization problem setup: The controller parameters are tuned such that the frequency response of the open-loop system becomes as close to the given desired shape as possible, while the closed-loop and controller stability constraints are satisfied. This tuning criteria are interpreted as the following minimization problem:

$$\min_K \sum_{k=1}^{N_{gain}} \left(\frac{|C(\omega_k) W_{plant}(j\omega_k)|}{W_{desired}(\omega_k)} - 1 \right)^2 \tag{3}$$

such that

$$\mathrm{Im}\left(\frac{C(j\phi_k) W_{plant}(j\phi_k)}{|C(j\phi_k) W_{plant}(j\phi_k)|} \Phi_k^{-1} \right) \geq 0 \ (k = 1, \cdots, N_{phase}) \tag{4}$$

$$a_i > 0 \ (i = 1, \cdots, \alpha_1), \ b_i, c_i > 0 \ (i = 1, \cdots, \alpha_2) \tag{5}$$

where $K \in \mathbb{R}^{11}$ is the controller parameter vector to be tuned, and $W_{plant}(j\omega_k)$ denotes the frequency response of the servo motor dynamics, which must be *measured* by the designer (Figure 3). Experimental data can be directly used, and no mathematical modelling is required. The objective function represents the error between actual and desired gain plots of the velocity open-loop system $(C(s)W_{plant}(s))$ at given frequencies.

The constraints (4) represent phase constraints on the open-loop system. $Im(x)$ denotes the imaginary part of x. Φ_k $(k = 1, \cdots, N_{phase})$ is a complex scalar of the absolute value 1, and of the phase angle representing the given phase constraint at the frequency ϕ_k. The constraints (4) assure that the phase angle at the frequency ϕ_k of the velocity open-loop system $(C(s)W_{plant}(s))$ is smaller than $\angle\Phi_k$.

The constraints (5) guarantee the stability of the controller itself. Notice that the constraints (5) are the sufficient condition for the stability of the controller (2), and therefore, may introduce slight conservativeness.

Solving the problem: The above problem is generally a nonconvex optimization problem, and thus it is not practical to try to find the global solution. In this paper, we simply employ the quasi-Newton local search algorithm to find a local solution. Although there is no guarantee that the global optimum is always found, it often shows sufficient search performance in many practical applications. In this example, we used a numerical optimization package, *Optimization Toolbox* [3] by the Mathworks Inc., on *MATLAB*. The command *fmincon* performs a local search using the sequential quadratic programming method for constrained nonlinear optimization problems [3].

2.3 Tuning Results

The computation time to solve the problem (3~5) was only 3.4 sec on a PC with Pentium III 1 GHz processor. Figure 7 compares frequency responses of the designed 5th order controller (denoted by $C_{new}(s)$) and the original PI controller (denoted by $C_{old}(s)$).

Both controllers were implemented on a 32-bit DSP board with the sampling time of 1.0 msec. Figure 8 shows measured frequency responses of the velocity open-loop system with each controller. It can be observed that $C_{new}(s)$ significantly reduces a resonance peak at about 250 rad/sec, without introducing much phase delay. Figure 9 compares measured frequency responses of the position closed-loop system (the transfer function from $r(s)$ to $y(s)$ in Figure 2). The resonance and counter-resonance peaks are almost completely cancelled by replacing the controller with $C_{new}(s)$. Recall that this example focuses only on the tuning of the velocity loop controller. As the position loop controller ($C_p(s)$ in Figure 2), a simple proportional gain is

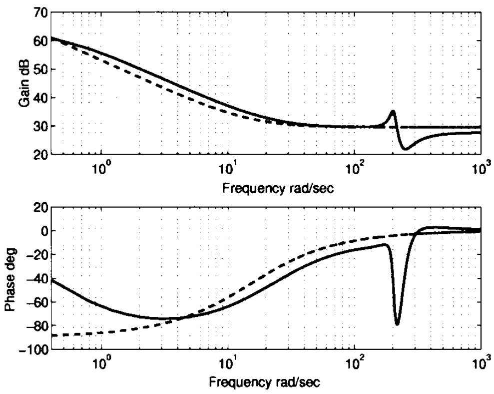

Figure 7. Frequency responses of controller dynamics (solid line: designed 5th order controller ($C_{new}(s)$), dashed: PI controller ($C_{old}(s)$))

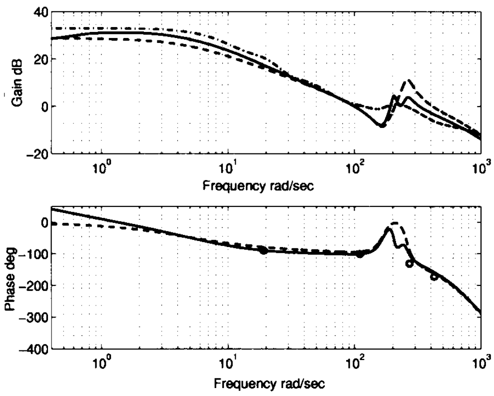

Figure 8. Frequency responses of velocity open-loop dynamics (solid: $C_{new}(s)$, dashed: $C_{old}(s)$). The dot-dashed line in the gain plot shows the "desired shape," $W_{desired}(\omega_k)$, and the circles in the phase plot represent phase constraints, $\Phi_k(\phi_k)$.

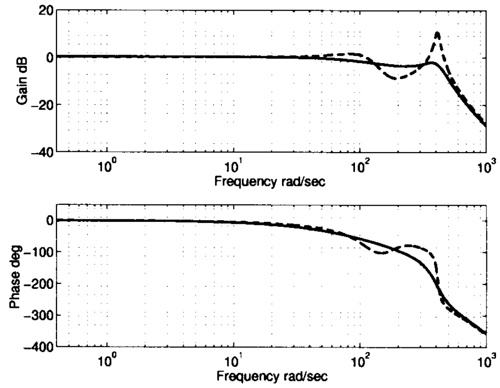

Figure 9. Frequency responses of position closed-loop dynamics (solid line: $C_{new}(s)$, dashed: $C_{old}(s)$)

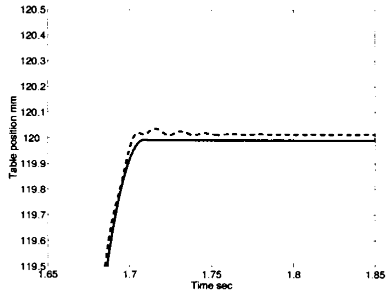

Figure 10. Time-domain step responses of the position closed-loop system (only the decelerating part is magnified) (solid line: $C_{new}(s)$, dashed: $C_{old}(s)$)

used, as is typical in commercial CNCs. In the experiments, the same position loop gain, $C_p(s) = 150$, was used in both $C_{old}(s)$ and $C_{new}(s)$ cases.

Figure 10 compares a time-domain step response of the position closed-loop system. When $C_{old}(s)$ is used, the vibration of the frequency corresponding to the resonance in the position closed-loop system can be observed. In order to reduce this vibration, one has to reduce the position loop gain and/or the gains of the velocity loop controller, which sacrifices the bandwidth of the closed-loop system. $C_{new}(s)$ successfully eliminated the vibration mode with the minimum sacrifice of the bandwidth.

3 Concluding Remarks

This paper presents a design methodology of a fixed-structure feedback controller based on the frequency-domain loop-shaping technique. It should be noted that the problem setup shown in Section 2.2 was inspired by the classical H_∞ loop-shaping design approach. Ibaraki and Tomizuka [4] presented an H_∞ optimization approach for fixed-structure controllers based on the frequency-domain loop-shaping setup. Compared to the celebrated H_∞ controller design, the proposed approach offers much more flexibility on the setup of design objectives and constraints. It does not even require a mathematical model of the plant dynamics. The drawback is that the present design methodology cannot mathematically guarantee the performance of the designed controller, and that the problem is not a convex optimization problem, unlike the H_∞ optimization cases. However, in many practical applications, it can be a more effective tool for a practical controller design than conventional optimal control theories.

Finally, it should be emphasized that the problem setup presented in Section 2.2 is merely an example. It can be changed flexibly depending on design requirements. In any cases, the designer is required to express controller design requirements in the form of "desired shape" of the system's frequency response plots.

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