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A NEW FORMULATION OF LASER STEP DIAGONAL MEASUREMENT TO IDENTIFY THREE-DIMENSIONAL VOLUMETRIC ERRORS

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ABSTRACT

The laser step diagonal measurement modifies the diagonal displacement measurement by executing a diagonal as a sequence of single-axis motions. It has been claimed that the step-diagonal test enables the identification of all the volumetric error components in the three-dimensional workspace, including linear errors, straightness and squareness errors, by only using a linear laser interferometer. This paper first discusses that an inherent issue with the conventional formulation of the step diagonal measurement is that setup errors of mirror and laser directions potentially impose a significant estimation error. Furthermore, when the machine's volumetric errors are unknown, it is generally not possible to completely eliminate setup errors. To address this issue, this paper proposes a new formulation of the laser step diagonal measurement, in order to accurately identify three-dimensional volumetric errors even under the existence of setup errors. The effectiveness of the proposed modified identification scheme is experimentally investigated by an application example of three-dimensional laser step diagonal measurements to a high-precision vertical machining center.
Key words: Step diagonal measurement, volumetric errors, machine tool, laser interferometer.

1 Introduction

In international or domestic standards such as the ISO standards, the motion accuracy of a machine tool is mostly evaluated by the axis-to-axis basis; the linear positioning error and the straightness error are evaluated for each axis, and the squareness error between two axes is evaluated. To ensure the motion accuracy over the entire workspace of a machine tool, it is important to evaluate all the three-dimensional components of the positioning error over the entire workspace [1]. Such error components are collectively called volumetric errors in this paper. Currently, ASME B5 (TC52) and ISO 230 (TC39) are working on the standardization of the definition of volumetric accuracy [2].

For the measurement of linear displacement errors, laser interferometers of the resolution sufficient to measure high-precision and ultra-precision machines are widely available in today's market. The measurement of other volumetric errors such as straightness and squareness errors is more difficult and time-consuming. Typically, straightness and squareness errors are measured by using a high-precision displacement sensor and an artifact such as a straight edge or a square edge. Naturally, the artifact whose geometric and dimensional accuracies are guaranteed to be higher than the accuracy of the measured machine is needed. Especially for the measurement of high-accuracy machine tools, it requires higher measurement cost. Furthermore, since the measurement is one-dimensional and its path is restricted to a line or a square, an operator must change the setup of a sensor and an artifact every time for the measurement of each different error component. For orthogonal three-axis machines, 3 linear displacement errors, 6 straightness errors and 3 squareness errors must be measured by different setups. Dual-beam laser systems or autocollimators to measure straightness and squareness errors are also available from many companies. They do not require an artifact such as a straight edge, but it is the same in that a different setup is needed to measure each different error component.

For quicker, lower-cost evaluation of volumetric errors of a machine tool, the standards ASME B5.54 [3] and ISO230-6 [4] define the diagonal measurement by using a laser interferometer. In the diagonal measurement, the machine moves along each of body diagonals of the machine's workspace in turn, and the diagonal displacement is measured by using a laser interferometer. Although the diagonal measurement can be considered as a good quick check of volumetric errors, it is clear that it cannot be used as a strict diagnosis of each volumetric error, as has been discussed in details by Chapman [5]. Under certain conditions, a machine can achieve a good result on diagonal tests, even though it has a poor volumetric accuracy. Furthermore, more importantly, it is impossible to distinguish the linear error, the straightness error, and the squareness error of each axis from the results of diagonal tests.

As an extension of the diagonal measurement, the step diagonal measurement, or the vector measurement, has been proposed by Wang [6; 7]. In the step diagonal measurement, each axis is moved one at a time along the "zig-zag" path toward the body diagonal direction. Figure 1 illustrates the setup of the 3D step-diagonal measurement. Wang claimed that additional data enables the identification of all the volumetric errors, namely the linear error, the straightness error, and the squareness error of each axis, from step diagonal measurements.

Our previously published paper [8] has discussed inherent issues with the volumetric error identification based on the conventional formulation of the two-dimensional version of step diagonal measurement. It showed that the formulation of the step diagonal measurement presented by Wang [6] is valid only when implicit assumptions related to laser and mirror setups are met, and that its inherent problem is that it is generally not possible to guarantee these conditions when volumetric errors of the machine are unknown. As a result, setup errors potentially impose significant errors on the identification of volumetric errors.

This paper presents the extension of this discussion to the three-dimensional (3D) version of laser step diagonal measurement. Then, to address this inherent issue, this paper will propose a new formulation of the 3D step diagonal measurement, such that each volumetric error in the 3D workspace can be identified from step diagonal measurements even when setup errors exist. The validity of the discussion on issues in step diagonal measurements and the effectiveness of the proposed modified identification scheme will be investigated experimentally by showing its application example to a commercial high-precision machining center.

2 Problem Statement and Conventional Formulation of Laser Step Diagonal Measurement

Figure 2 illustrates the setup of the 3D step diagonal measurement when there is only one block. As the machine spindle, where a plane mirror is attached, moves along a "zig-zag" path, the moving distance along the body diagonal is measured by using a laser interferometer. Suppose that the laser is aligned to the body diagonal AG, as is illustrated in Fig. 2. Suppose that this direction is represented by the unit vector $l_{ppp} = \left[\, l_{x,ppp},\ l_{y,ppp},\ l_{y,ppp} \right]$ (this setup is referred to as ppp measurement hereafter).

Define $E_x(x)$, $E_y(x)$ and $E_z(x)$ as the positioning error in x-, y-, and z-directions, respectively, due to the motion toward x direction (i.e. A→B). $E_x(y)$, $E_y(y)$, $E_z(y)$, $E_x(z)$, $E_y(z)$, and $E_z(z)$ are defined similarly. In this paper, these total nine error components are called volumetric errors. The distance measured by a laser interferometer with the motion toward x (A→B), y (B→C), and z (C→G) directions are respectively given by $R_{x,ppp}$, $R_{y,ppp}$, and $R_{z,ppp}$. A similar measurement is done as the laser is aligned along body diagonals BH and DF (these setups are respectively referred to as npp and pnp measurements). $R_{x,npp}$, $R_{y,npp}$, $R_{z,npp}$, $R_{x,pnp}$, $R_{y,pnp}$, and $R_{z,pnp}$ are defined similarly. Then, we have [6]:

$$
\begin{bmatrix}
l_{x,ppp} & l_{y,ppp} & l_{z,ppp} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & l_{x,ppp} & l_{y,ppp} & l_{z,ppp} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{x,ppp} & l_{y,ppp} & l_{z,ppp} \\
-l_{x,npp} & -l_{y,npp} & -l_{z,npp} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & l_{x,npp} & l_{y,npp} & l_{z,npp} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{x,npp} & l_{y,npp} & l_{z,npp} \\
l_{x,pnp} & l_{y,pnp} & l_{z,pnp} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -l_{x,pnp} & -l_{y,pnp} & -l_{z,pnp} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{x,pnp} & l_{y,pnp} & l_{z,pnp}
\end{bmatrix}
\cdot
\begin{bmatrix}
a+E_x(x) \\ E_y(x) \\ E_z(x) \\ E_x(y) \\ a+E_y(y) \\ E_z(y) \\ E_x(z) \\ E_y(z) \\ a+E_z(z)
\end{bmatrix}
=
\begin{bmatrix}
R_{x,ppp} \\ R_{y,ppp} \\ R_{z,ppp} \\ R_{x,npp} \\ R_{y,npp} \\ R_{z,npp} \\ R_{x,pnp} \\ R_{y,pnp} \\ R_{z,pnp}
\end{bmatrix}
\tag{1}
$$

Assume nominal laser directions, i.e.:

$$
\begin{aligned}
\left[\, l_{x,ppp}\ l_{y,ppp}\ l_{z,ppp} \right] &= \frac{1}{\sqrt{3}} \left[\, 1\ 1\ 1 \right] \\
\left[\, l_{x,npp}\ l_{y,npp}\ l_{z,npp} \right] &= \frac{1}{\sqrt{3}} \left[\, -1\ 1\ 1 \right] \\
\left[\, l_{x,pnp}\ l_{y,pnp}\ l_{z,pnp} \right] &= \frac{1}{\sqrt{3}} \left[\, 1\ -1\ 1 \right]
\end{aligned}
\tag{2}
$$

Then, all the volumetric errors, $E_x(x)$, $\cdots$, $E_z(z)$ can be estimated from measured diagonal distances, $R_{x,ppp}$, $\cdots$, $R_{z,pnp}$ by solving Eq. (1).

For the simplicity, the present formulation assumed the one-block case illustrated in Fig. 2. In the case with N blocks, define, for example, $E_x(x(k))$ $(k = 1, \cdots, N)$ as the positioning error in the X direction, when the machine moves toward the X direction from the reference position $x(k-1)$ to $x(k)$. All the other variables are defined analogously. Then, the formulation (1) can be extended in a straightforward manner to

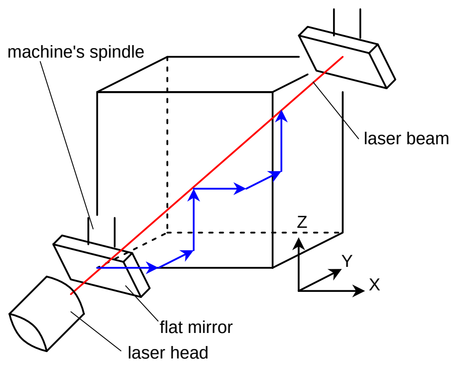


Figure 1. Schematics of 3D laser step diagonal measurement

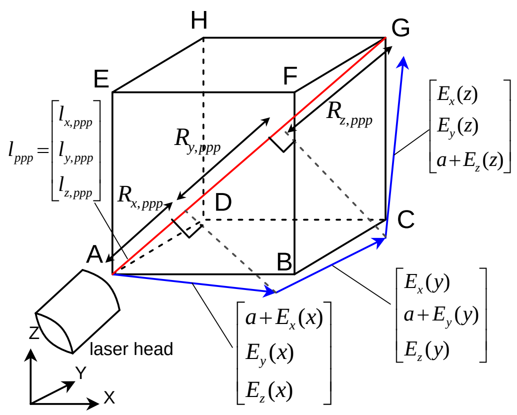


Figure 2. Volumetric errors and diagonal displacements (single block case)

estimate volumetric errors, $E_x(x(k))$, $\cdots$, $E_z(z(k))$ from measured diagonal distances, $R_{x,ppp}(k)$, $\cdots$, $R_{z,pnp}(k)$ $(k = 1, \cdots, N)$.

3 Issues in Conventional Formulation of Laser Step Diagonal Measurement

3.1 Issues in Conventional Formulation

In our previous paper [8], we have discussed inherent issues with the volumetric error identification based on the conventional formulation of the 2D version of step diagonal measurement. The same discussion can be applied to the 3D version of the conventional formulation of step diagonal measurement presented in the previous section. This section briefly reviews these issues.

First, notice that Eq. (1) is valid only when the following conditions are satisfied: 1) Laser beam directions must be precisely aligned to nominal directions, and 2) The flat mirror must be precisely aligned perpendicular to the laser beam direction. An inherent problem with the conventional formulation (1) is that when the conditions above are not met, setup errors of laser and mirror directions potentially impose a significant error on the estimates of volumetric errors. Furthermore, *since the direction of the laser beam and the flat mirror can be only aligned based on the motion of the machine to be measured*, it is generally not possible to guarantee the satisfaction of the conditions (1) and (2), when volumetric errors of the machine are unknown.

(1) Misalignment of laser beam directions

In practice, the laser beam direction can be only aligned based on the motion of the machine to be measured. That is, in a typical setup, the laser beam direction is aligned such that it becomes parallel to the machine's diagonal. For example, when the machine moves from A to G in Fig. 2, the laser direction is adjusted such that the deviation of the location of the laser spot on the mirror is minimized (this alignment can be done more precisely by using a quad-detector). Here, when the machine has volumetric errors and they are unknown, it is not possible to align the laser beam perfectly to the nominal direction, no matter how careful an operator sets up the laser beam direction.

An illustrative example is shown in Fig. 3 (for the simplicity, this example shows the 2D case. Exactly the same discussion is possible for the 3D case). This example assumes that $E_y(y) > 0$, $E_x(x) = E_y(x) = E_x(y) = 0$. By carefully setting up laser directions, the laser direction can be ideally aligned to the machine's diagonal. However, since the machine's diagonals are not perpendicular to each other due to volumetric errors, laser directions in pp- and np- measurements cannot be perpendicular to each other.

(2) Misalignment of mirror directions

Similarly, the direction of the flat mirror can be only aligned based on the motion of the machine to be measured. Typically, the mirror direction is adjusted as follows: when the spindle (where the flat mirror is attached) moves in the direction perpendicular to the diagonal direction, the mirror direction is adjusted such that the measured diagonal distance becomes approximately equal at both ends of the mirror. Similarly as above, when the machine's volumetric errors are unknown, it is not possible to align the mirror perfectly normal to the laser direction.

Fig. 4 illustrates the case where $E_y(y) > 0$, $E_x(x) = E_y(x) = E_x(y) = 0$. In such a case for example, the mirror can be aligned to the diagonal direction of the machine's motion. It does not, however, mean that the mirror direction is perpendicular to the laser direction.

Note that angular errors with the machine's motion toward X, Y, and Z directions also affect diagonal displacements. Soons [9] and Yang et al. [10] gave the formulation of the effect of angular errors on identification accuracies for the 3D laser step diagonal measurement. In this paper, under the assumption that angular errors are negligibly small compared to

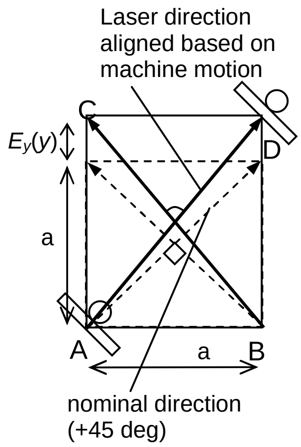


Figure 3. Misalignment of laser beam direction caused by machine's volumetric error.

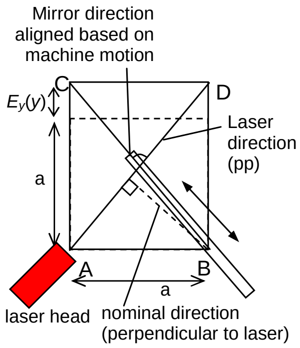


Figure 4. Misalignment of mirror direction caused by machine's volumetric error.

positioning errors, the effect of angular errors is not considered.

3.2 Sensitivity of Setup Errors

To quantitatively evaluate the effect of aforementioned setup errors on the estimation accuracy of laser step diagonal measurements and to further clarify critical issues with the conventional formulation, this section presents the sensitivity analysis of setup errors on the estimates of volumetric errors. The misalignment errors of laser and mirror directions are referred to as setup errors hereafter.

First, notice that setup errors potentially impose significant effect on the measured diagonal displacement when the mirror center is not on the laser axis, while its effect becomes negligibly small when the mirror center is on the laser axis. For example, in the ppp measurement as shown in Fig. 2, when the mirror center is at B or C, both laser and mirror misalignment errors may cause significant error on $R_{x,ppp}$ or $R_{y,ppp}$. However, their effect on the diagonal distance, $R_{x,ppp}+R_{y,ppp}+R_{z,ppp}$, is negligibly small. In the k-th block, suppose that the "nominal" step diagonal distance for the x-motion is denoted by $\tilde{R}_{x(k),ppp}$, when the laser direction is perfectly aligned to the nominal direction and the mirror is aligned perfectly perpendicular to the laser direction. The effect of the misalignment of laser and mirror directions can be given in the following form:

$$R_{x(k),ppp} = \tilde{R}_{x(k),ppp} + \delta R_{x,ppp} \quad (3)$$

where $\delta R_{x,ppp}$ is the diagonal displacement error caused by setup errors. Under the assumption that the machine's angular error is negligibly small, we can approximate that the effect of setup errors, $\delta R_{x,ppp}$, is the same for all the blocks. Other parameters, $\delta R_{y,ppp}$, $\delta R_{z,ppp}$, $\delta R_{x,npp}$, $\delta R_{y,npp}$, $\delta R_{z,npp}$, $\delta R_{x,pnp}$, $\delta R_{y,pnp}$, $\delta R_{z,pnp}$, are defined analogously. The discussion above indicates that the following approximation holds:

$$\begin{aligned} \delta R_{x,ppp} + \delta R_{y,ppp} + \delta R_{z,ppp} &\approx 0 \\ \delta R_{x,npp} + \delta R_{y,npp} + \delta R_{z,npp} &\approx 0 \\ \delta R_{x,pnp} + \delta R_{y,pnp} + \delta R_{z,pnp} &\approx 0 \end{aligned} \quad (4)$$

By solving Eq.(1) (its the extension to the multiple blocks case) for estimated volumetric errors in the k-th block, we have:

$$\hat{E}_x(x(k)) = \frac{\sqrt{3}}{2}\left\{R_{x(k),ppp} + R_{x(N-k+1),npp} - (\delta R_{x,ppp} + \delta R_{x,npp})\right\} - a \quad (5)$$

$$\hat{E}_y(x(k)) = \frac{\sqrt{3}}{2}\left\{R_{x(k),ppp} - R_{x(k),pnp} - (\delta R_{x,ppp} - \delta R_{x,pnp})\right\} \quad (6)$$

$$\hat{E}_z(x(k)) = \frac{\sqrt{3}}{2}\left\{R_{x(k),pnp} - R_{x(N-k+1),npp} - (\delta R_{x,pnp} - \delta R_{x,npp})\right\} \quad (7)$$

$$\hat{E}_x(y(k)) = \frac{\sqrt{3}}{2}\left\{R_{y(k),ppp} - R_{y(k),npp} - (\delta R_{y,ppp} - \delta R_{y,npp})\right\} \quad (8)$$

$$\hat{E}_y(y(k)) = \frac{\sqrt{3}}{2}\left\{R_{y(k),ppp} + R_{y(N-k+1),pnp} - (\delta R_{y,ppp} + \delta R_{y,pnp})\right\} - a \quad (9)$$

$$\hat{E}_z(y(k)) = \frac{\sqrt{3}}{2}\left\{R_{y(k),npp} - R_{y(N-k+1),pnp} - (\delta R_{y,npp} - \delta R_{y,pnp})\right\} \quad (10)$$

$$\hat{E}_x(z(k)) = \frac{\sqrt{3}}{2}\left\{R_{z(k),ppp} - R_{z(k),npp} - (\delta R_{z,ppp} - \delta R_{z,npp})\right\} \quad (11)$$

$$\hat{E}_y(z(k)) = \frac{\sqrt{3}}{2}\left\{R_{z(k),ppp} - R_{z(k),pnp} - (\delta R_{z,ppp} - \delta R_{z,pnp})\right\} \quad (12)$$

$$\hat{E}_z(z(k)) = \frac{\sqrt{3}}{2}\left\{R_{z(k),npp} + R_{z(k),pnp} - (\delta R_{z,npp} + \delta R_{z,pnp})\right\} - a \quad (13)$$

The nominal laser beam directions (3) are assumed here. Equations (5)$\sim$(13) indicate that the estimated volumetric errors are subject to the influence of setup errors by a factor of $\frac{\sqrt{3}}{2}$. Furthermore, it can be easily observed that it is not possible to identify nine setup errors, $\delta R_{x,ppp}, \cdots, \delta R_{z,pnp}$ from laser step diagonal measurements.

4 New Formulation of Laser Step Diagonal Measurement

In this section, we present a new formulation of laser step diagonal measurement such that setup errors do not impose any effect on estimated volumetric errors, and thus the machine's volumetric errors can be accurately estimated even when there exist significant setup errors in both laser and mirror alignments.

As has been discussed in the previous section, it is not possible to identify nine setup errors, $\delta R_{x,ppp}, \cdots, \delta R_{z,pnp}$ in Eqs. (5)∼(13) from laser step diagonal measurements. In order to cancel setup errors, a remedy is to directly measure linear error components, $E_x(x(k))$, $E_y(y(k))$ and $E_z(z(k))$ ($k = 1, \cdots, N$). By directly measuring linear positioning errors, the effect of setup errors can be separately identified. Therefore, all the other error components (namely, error components in the direction normal to the feed direction) can be identified even under the existence of significant setup errors.

First, for the simplicity of notation, define

$$\begin{aligned} \lambda_{xx} &:= \delta R_{x,ppp} + \delta R_{x,npp}, \ \ \lambda_{yx} := \delta R_{x,ppp} - \delta R_{x,pnp}, \\ \lambda_{zx} &:= \delta R_{x,pnp} - \delta R_{x,npp}, \ \ \lambda_{xy} := \delta R_{y,ppp} - \delta R_{y,npp}, \\ \lambda_{yy} &:= \delta R_{y,ppp} + \delta R_{y,pnp}, \ \ \lambda_{zy} := \delta R_{y,npp} - \delta R_{y,pnp}, \\ \lambda_{xz} &:= \delta R_{z,ppp} - \delta R_{z,npp}, \ \ \lambda_{zy} := \delta R_{z,ppp} - \delta R_{z,pnp}, \\ \lambda_{zz} &:= \delta R_{z,npp} + \delta R_{z,pnp} \end{aligned} \tag{14}$$

which represent the effect of setup errors in Eqs. (5)∼(13), respectively. Then, when linear positioning errors, $E_x(x(k))$, $E_y(y(k))$, $E_z(z(k))$ ($k = 1, \cdots, N$), are measured, from Eqs. (5), (9), and (13), λ_{xx}, λ_{yy}, and λ_{zz} can be respectively estimated as follows:

$$\hat{\lambda}_{xx} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\left(E_x(x(k))+a\right)+\left(R_{x(k),ppp}+R_{x(N-k+1),npp}\right)\right\} \tag{15}$$

$$\hat{\lambda}_{yy} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\left(E_y(y(k))+a\right)+\left(R_{y(k),ppp}+R_{y(N-k+1),pnp}\right)\right\} \tag{16}$$

$$\hat{\lambda}_{zz} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\left(E_z(z(k))+a\right)+\left(R_{z(k),npp}+R_{z(k),pnp}\right)\right\} \tag{17}$$

Notice that, to define the position in the 3D space, the orientation of the coordinate system can be arbitrarily set. For example, the coordinate system can be defined such that:

$$\text{mean}\left\{\hat{E}_y(x(k))\right\} = \text{mean}\left\{\hat{E}_z(x(k))\right\} = \text{mean}\left\{\hat{E}_y(z(k))\right\} = 0 \tag{18}$$

Under this assumption, three more parameters, namely, λ_{yx}, λ_{zx}, and λ_{yz}, can be estimated from Eqs. (6)(7)(12) as follows:

$$\hat{\lambda}_{yx} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\hat{E}_y(x(k))+\left(R_{x(k),ppp}-R_{x(k),pnp}\right)\right\} \tag{19}$$

$$\hat{\lambda}_{zx} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\hat{E}_z(x(k))+\left(R_{x(k),pnp}-R_{x(N-k+1),npp}\right)\right\} \tag{20}$$

$$\hat{\lambda}_{yz} := \text{mean}\left\{-\frac{2}{\sqrt{3}}\hat{E}_y(z(k))+\left(R_{z(k),ppp}-R_{z(k),pnp}\right)\right\} \tag{21}$$

Now, from the definitions (14) and Eq. (4), we have:

$$\hat{\lambda}_{xy} = -\hat{\lambda}_{yx} - \hat{\lambda}_{zx} - \hat{\lambda}_{xz} \tag{22}$$

$$\hat{\lambda}_{zy} = -\hat{\lambda}_{yx} - \hat{\lambda}_{yz} - \hat{\lambda}_{xy} \tag{23}$$

$$\hat{\lambda}_{xz} = -\hat{\lambda}_{zx} - \hat{\lambda}_{xx} - \hat{\lambda}_{yy} - \hat{\lambda}_{zz} \tag{24}$$

From Eqs. (15)(16)(17), Eqs. (19)(20)(21), and Eqs. (22)∼(24), we can identify all of nine parameters, $\hat{\lambda}_{xx}, \hat{\lambda}_{yx}, \hat{\lambda}_{yz}, \hat{\lambda}_{xy}, \hat{\lambda}_{yy}, \hat{\lambda}_{zy}, \hat{\lambda}_{xz}, \hat{\lambda}_{yz}, \hat{\lambda}_{zz}$.

When they are identified, the volumetric errors can be identified by Eqs. (5)∼(13).

Remark: In the two-dimensional case discussed in [8], setup errors only affect the estimates of linear error components, $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$. In other words, when $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$ are replaced with the measured values, the conventional formulation (two-dimensional version of Eq. (1)) can give a good estimate for normal error components, $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$ ($k = 1, \cdots, N$). This does not hold for the three-dimensional case. As has been discussed above, the conventional formulation (1) may give a significant estimation error for normal error components, even when linear error components are directly measured and excluded from the estimation.

5 Experimental validation

The problems with the conventional formulation of laser step diagonal measurement, and the effectiveness of its proposed formulation, are experimentally validated by an application example to a three-axis vertical-type high-precision commercial machining center.

The machine has three orthogonal linear axes, which are all driven by a ball screw and a servo motor with a slide guideway. Its positioning resolution is 0.1 μm in all the axes. The machine's strokes are: X: 900mm, Y: 500mm, Z: 350mm. For laser measurements, a laser doppler displacement meter, MCV-500 by Optodyne, Inc. is used. Laser beam directions are aligned by using a quad-detector, LD42 by Optodyne, Inc. The step diagonal measurements are done with the step size $a = 10$ mm, over the entire range of 120 mm × 120 mm × 120 mm (i.e. 12 blocks in X, Y, and Z directions).

First, volumetric errors are estimated by the conventional formulation using step diagonal measurements only. Figures 5∼7 show estimated positioning errors with respect to the reference position ("Estimated by conventional formulation"). Note that the positioning error with respect to the reference position is defined by the accumulation of error components defined in Section 2. Namely:

$$\begin{bmatrix} \hat{\varepsilon}_x(x(k)) \\ \hat{\varepsilon}_y(x(k)) \\ \hat{\varepsilon}_z(x(k)) \end{bmatrix} = \sum_{i=1}^{k} \begin{bmatrix} \hat{E}_x(x(i)) \\ \hat{E}_y(x(i)) \\ \hat{E}_z(x(i)) \end{bmatrix}, \quad \begin{bmatrix} \hat{\varepsilon}_x(y(k)) \\ \hat{\varepsilon}_y(y(k)) \\ \hat{\varepsilon}_z(y(k)) \end{bmatrix} = \sum_{i=1}^{k} \begin{bmatrix} \hat{E}_x(y(i)) \\ \hat{E}_y(y(i)) \\ \hat{E}_z(y(i)) \end{bmatrix}$$

$$\begin{bmatrix} \hat{\varepsilon}_x(z(k)) \\ \hat{\varepsilon}_y(z(k)) \\ \hat{\varepsilon}_z(z(k)) \end{bmatrix} = \sum_{i=1}^{k} \begin{bmatrix} \hat{E}_x(z(i)) \\ \hat{E}_y(z(i)) \\ \hat{E}_z(z(i)) \end{bmatrix} \quad (25)$$

For each of step diagonal measurement, the same measurement is repeated by five times. Figures 5∼ 7 plot the mean of estimated errors by the marks "∘", as well as their variation at each measurement point by horizontal parallel lines ("=").

For the comparison, linear positioning errors in X, Y, and Z directions, $\varepsilon_x(x(k))$, $\varepsilon_y(y(k))$, and $\varepsilon_z(z(k))$, were measured by using the same laser interferometer aligned directly toward X, Y, and Z directions, respectively. The straightness errors in X, Y, and Z directions were measured by using a laser displacement sensor, LK-G10 by Keyence Corp. (measurement resolution: 0.01 μm), and an optical flat as the straight-edge (according to the manufacturer's calibration chart, its straightness error is $<$ PV $\lambda/4$, $\lambda = 0.6328\mu$m). The squareness errors of X-Y, Y-Z, X-Z axes were measured by using the same laser displacement sensor, LK-G10 by Keyence Corp., and the square-edge by Fujita Works, Ltd. (according to the manufacturer's calibration chart, its squareness error is -0.5 μm / 150 mm). The error components in the direction normal to the feed directions, $\varepsilon_y(x(k))$, $\varepsilon_z(x(k))$, $\cdots$, can be computed by combining measured straightness and squareness errors. In Figs. 5∼ 7, these measured values are also plotted ("Measured by using artefact"). Similarly as the estimated values, the same measurement was repeated by five times, and their mean values as well as their variation are plotted in the figures.

Then, volumetric errors are estimated based on the proposed formulation of step diagonal measurements presented in Section 4, by using displacement profiles in ppp-, npp-, pnp-, X-, Y-, and Z- directions, measured by using the same laser doppler displacement meter, MCV-500. Estimated profiles are also plotted in Figs. 5∼7 ("Estimated by proposed formulation"). Note that in all the cases, the coordinate system is defined as shown in Eq. (18).

Table 1 summarizes measured and estimated straightness and squareness errors. Here, the straightness error is defined by the maximum variation of mean values of normal errors (for example, $\varepsilon_y(x(k))$ for the straightness error of X axis to the Y direction) from their least-square mean line. The squareness errors are defined by the gradient of the least-square mean line of $\varepsilon_x(y(k))$ (X-Y), $\varepsilon_x(z(k))$ (X-Z), and $\varepsilon_z(y(k))$ (Y-Z) with respect to that of $\varepsilon_y(x(k))$, $\varepsilon_z(x(k))$, and $\varepsilon_y(z(k))$, respectively.

From Figs. 5∼7 and Table 1, it can be clearly observed that, when the conventional formulation (1) is used, the estimated volumetric errors from step diagonal measurements have significant estimation errors, although an experienced operator set up laser and mirror directions very carefully in the experiments. The estimation error is particularly large in the estimates of linear positioning errors in Y and Z directions (at maximum about 22 μm over 120mm in the Z direction).

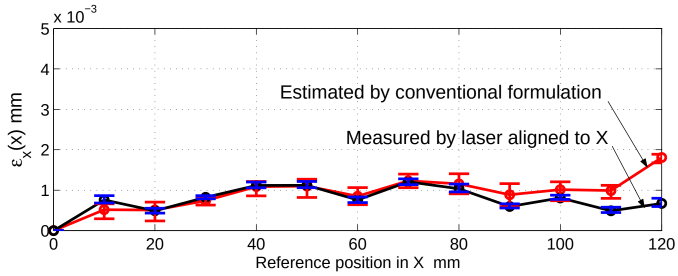
In overall, volumetric errors of the measured machine are smaller than typical general-purpose machining centers in the market. The straightness errors of X, Y, and Z axes are all smaller than 1 μm. Considering the measurement uncertainties associated with the laser doppler displacement meter or the artefacts, it is difficult to draw any conclusion from the comparison in straightness errors. The squareness errors of the measured machine are relatively larger. Fig. 6(a) (→ the squareness of X-Y), Fig. 7(a) (→ the squareness of X-Z), and Fig. 7(b) (→ the squareness of Y-Z), show that the conventional formulation of step diagonal measurements results in larger estimation errors (see also Table 1). For example, as is shown in Fig. 6(a), the measured squareness error between X and Y axes is -1.4 μm / 120 mm. The conventional formulation of step diagonal measurements gives its estimate of 3.2 μm / 120 mm. The estimates given by applying the proposed formulation of step diagonal measurements show better match with the measured values. In the case of the squareness error between X and Y, its estimate based on the proposed formulation is -1.2 μm / 120 mm.

6 Conclusion

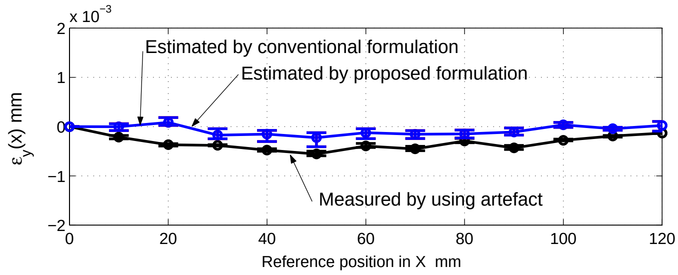
The conventional formulation of the step diagonal measurement proposed by Wang [6] is valid only when the following implicit conditions are met: (1) laser beam directions are precisely aligned to nominal directions, and (2) the flat mirror is precisely aligned perpendicular to the laser beam direction. An inherent problem with the conventional formulation of the step diagonal measurement is that it is generally not possible to meet these conditions by the adjustment of the setup, when volumetric errors of the machine are unknown. This paper first presented the quantitative analysis of the effect of setup errors on estimated volumetric errors by 3D laser step diagonal measurements. It was shown that setup errors may impose a significant effect on the estimates of volumetric errors. Therefore, in general cases where setup errors cannot be completely eliminated, it is not possible to accurately identify all the volumetric errors by using the conventional formulation of laser step diagonal measurement.

Then, this paper proposed the new formulation of 3D laser step diagonal measurements. When linear positioning errors are directly measured, the effects of setup errors on step diagonal measurements can be separately identified by using the proposed formulation. Therefore, all the volumetric errors can be identified even under the existence of setup errors.

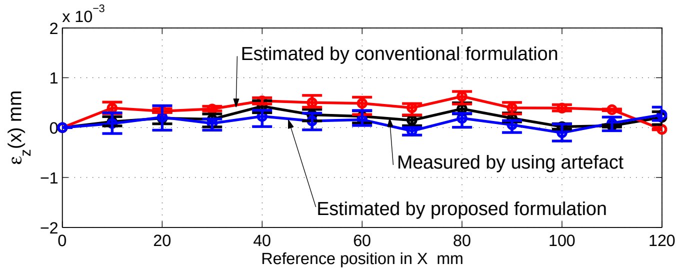
As an application example, the proposed scheme was applied to estimate 3D volumetric errors on a high-precision machining center of the positioning resolution of 0.1 μm. Experimental results indicated that the proposed formulation resulted in much smaller estimation errors than those by the conventional formulation. Based on the proposed formulation, the



(a) The positioning error in X with the motion toward X, $\varepsilon_x(x(k))$

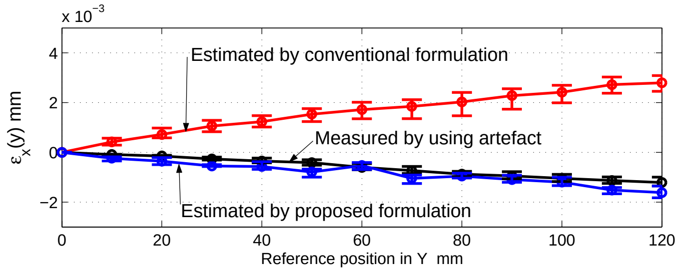


(b) The positioning error in Y with the motion toward X, $\varepsilon_y(x(k))$

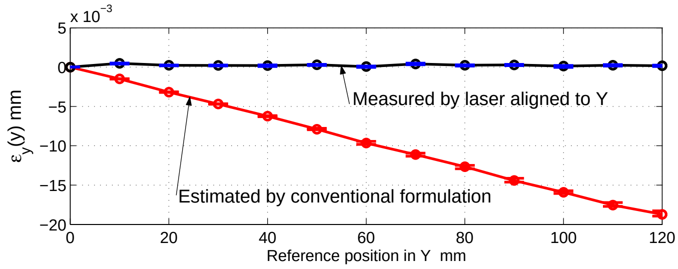


(c) The positioning error in Z with the motion toward X, $\varepsilon_z(x(k))$

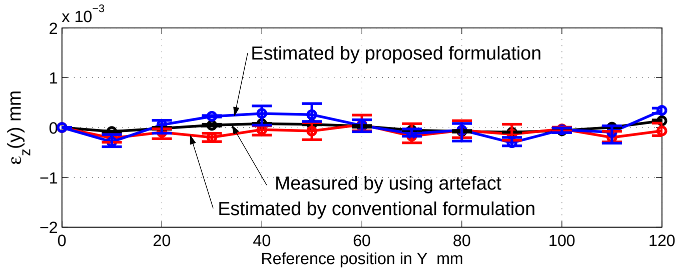
Figure 5. Measured and identified volumetric errors for the motion toward X direction.



(a) The positioning error in X with the motion toward Y, $\varepsilon_x(y(k))$



(b) The positioning error in Y with the motion toward Y, $\varepsilon_y(y(k))$



(c) The positioning error in Z with the motion toward Y, $\varepsilon_z(y(k))$

Figure 6. Measured and identified volumetric errors for the motion toward Y direction.

squareness error of X-Y, X-Z, and Y-Z axes were estimated with an estimation error of at maximum about 3 μm.

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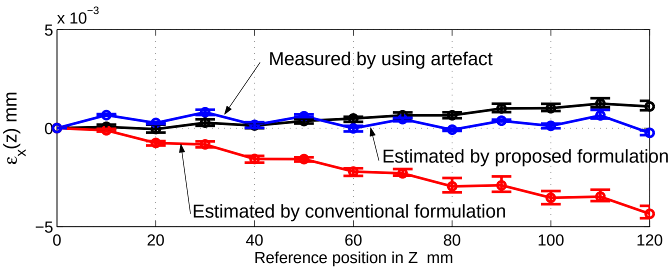
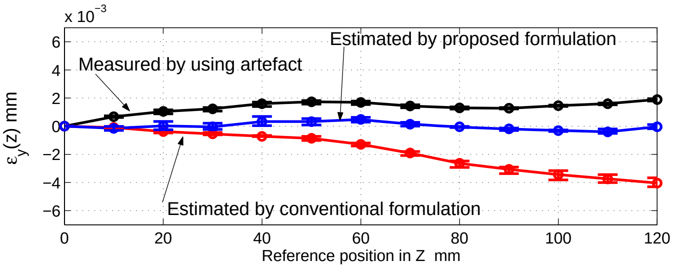
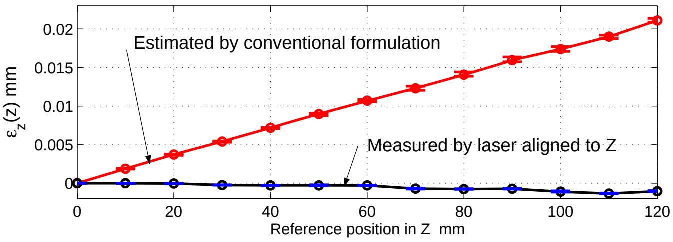


(a) The positioning error in X with the motion toward Z, $\varepsilon_x(z(k))$



(b) The positioning error in Y with the motion toward Z, $\varepsilon_y(z(k))$



(c) The positioning error in Z with the motion toward Z, $\varepsilon_z(z(k))$

Figure 7. Measured and identified volumetric errors for the motion toward Z direction.

Table 1. Measured and estimated straightness and squareness errors.

	Measured	Conventional estimation	Proposed estimation
Positioning error in X	0.8 μm	1.7 μm	–
Positioning error in Y	0.2 μm	-18.3 μm	–
Positioning error in Z	-1.1 μm	20.9 μm	–
Straightness of X axis (Y direction)	0.5 μm	0.4 μm	0.4 μm
Straightness of X axis (Z direction)	0.4 μm	0.7 μm	0.4 μm
Straightness of Y axis (X direction)	0.1 μm	0.6 μm	0.5 μm
Straightness of Y axis (Z direction)	0.2 μm	0.3 μm	0.7 μm
Straightness of Z axis (X direction)	0.3 μm	0.6 μm	0.8 μm
Straightness of Z axis (Y direction)	0.8 μm	1.1 μm	0.9 μm
Squareness of X-Y	-1.2 μm	3.2 μm	-1.4 μm
Squareness of X-Z	1.2 μm	-4.3 μm	0.1 μm
Squareness of Y-Z	2.7 μm	-3.9 μm	-0.1 μm

∗ All the errors are over the range of 120 mm.