

# Estimation of Three-dimensional Volumetric Errors of Numerically Controlled Machine Tools by a Laser Tracker

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## Abstract

This paper describes the estimation of three-dimensional volumetric errors of a machining center by using laser trackers. A laser tracker is a laser interferometer with the mechanism to steer the laser direction to follow a target retroreflector. Based on the triangulation principle, the three-dimensional position of the target can be estimated from measured laser displacements. We evaluate its estimation accuracy by experiments and simulations.

**Key words:** Laser tracker, volumetric errors, estimation accuracy

## 1. Introduction

In recent years, the importance of the evaluation of three-dimensional volumetric accuracy of machine tools has been more widely recognized [1]. In order to improve the machining accuracy of large and complex parts, such as parts of airplanes, it is necessary to reduce three-dimensional volumetric errors over the entire movable range of machine tools [2]. However, no measuring instrument is available to directly and efficiently measure volumetric errors over the entire workspace.

This paper uses a laser tracker to directly measure volumetric errors. A laser tracker is a laser interferometer with the mechanism to steer the laser direction to follow a target retroreflector. Although a laser tracker can measure only the distance between the tracker and the target, three-dimensional positions of the target can be estimated based on the triangulation principle.

This paper discusses the application of the laser tracking system to the calibration of volumetric errors of a machining center. Our study uses the laser tracker developed by a part of authors in National Institute of Advanced Industrial Science and Technology [3]. In our previous work [4], its measurement range was not sufficiently large (40mm × 40mm × 40mm) and its

estimation accuracy of the machine's volumetric errors was not sufficient (estimation errors up to 20 to 50μm). This paper will further present our effort to expand the measured range and to improve its estimation accuracy. Potential causes of its estimation error are studied by simulations.

## 2. The Laser Tracking System

### 2.1 The mechanism of Laser Tracker

The mechanism of the laser tracker is illustrated in Fig. 1 [5]. An outlook of the laser tracker in this paper is shown in Fig. 2 [5]. It consists of a steering mirror to control the laser direction and a quadrant photo diode to locate the laser spot returned from the target. When the laser beam reaches a corner cube, a target attached to the machine's spindle, it returns back to the tracker. The principle of tracking is shown in Fig. 3. The laser tracker can follow the target by regulating the orientation of the moving mirror such that the offset between the incident beam and the reflected beam is regulated.

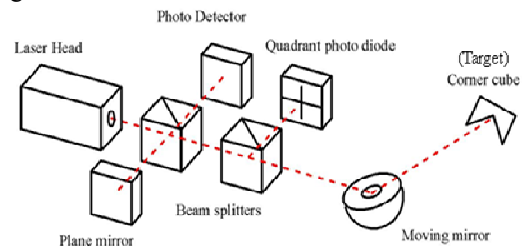


Fig. 1 Optical system of the laser tracker

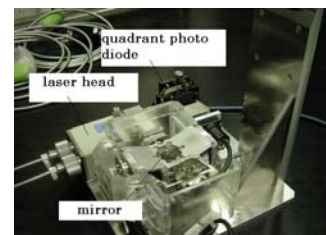


Fig. 2 Outlook of the laser tracker

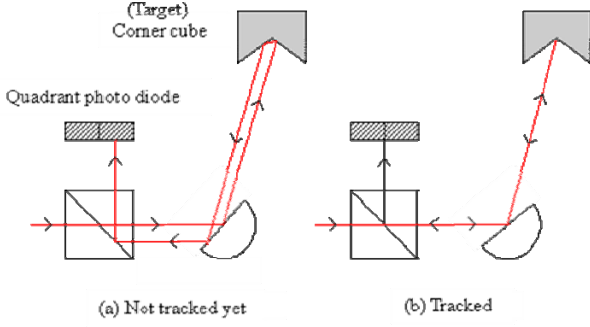


Fig. 3 Principle of tracking

## 2.2 Algorithm to Calculate Target Position

The position of the target can be calculated by measuring the distance to it from three different tracker locations based on the triangulation principle, when each tracker location is exactly known. In practice, however, it is not possible to measure the exact position of the tracker. To solve this issue, tracker locations are typically self-calibrated by using redundant measurement from the fourth location of the laser tracker. This section explains an algorithm to calculate the target position and tracker locations from four measurements from different tracker locations.

The coordinate system is defined such that four tracker locations are given by  $U_j = [U_j \ V_j \ W_j]^T$  ( $j=1,2,3,4$ ). Suppose that target locations are given by  $u_i = [u_i \ v_i \ w_i]^T$  ( $i=1,2,\dots,m$ ), where  $m$  represents the number of locations. Define the vector,  $p$ , that contains all the unknown parameters.  $p$  is given by:

$$p = [U_2 \ U_3 \ V_3 \ U_4 \ V_4 \ W_4 \ u_1^T \ u_2^T \ \dots \ u_m^T]^T$$

The coordinate system and unknown parameters are shown in Fig. 4.

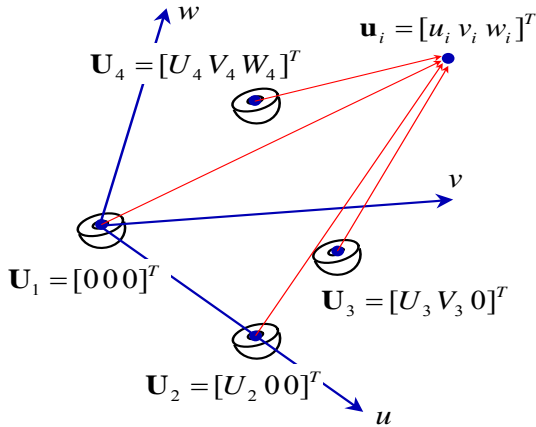


Fig. 4 The coordinate system and unknown parameters

The laser displacement,  $d_{ij}$ , measured by the  $j$ -th tracker to the  $i$ -th target location is given by:

$$d = [d_{11} \ d_{21} \ d_{31} \ d_{41} \ \dots \ d_{m1} \ d_{m2} \ d_{m3} \ d_{m4}]^T$$

The relation between  $d_{ij}$  and target and tracker positions can be expressed in the following equation:

$$d_{ij} = |u_i - U_j| - |u_1 - U_j| \equiv f_{ij} \quad (1)$$

The relation between  $d$  and  $p$  can be written in the

following equation:

$$d - f(p) = 0 \quad (2)$$

The equation above contains  $6+3m$  unknown parameters. Notice that the number of laser measurements is  $4m$ . Therefore, when  $m > 6$ , the number of measurements exceeds the number of unknown parameters. Unknown parameters,  $p$ , can be identified such that  $|d - f(p)|$  is minimized. It can be solved by applying the Newton method with linearizing the function  $f(p)$ . At the  $k$ -th step,

$$d - f(p) \cong d - \left( f(p^{(k)}) + \left( \frac{\partial f}{\partial p} \Big|_{p=p^{(k)}} \right) \cdot (p - p^{(k)}) \right) = 0 \quad (3)$$

Define:

$$\Delta d^{(k)} = d - f(p^{(k)}), \quad \Delta p^{(k)} = p - p^{(k)}, \quad A^{(k)} = \frac{\partial f}{\partial p} \Big|_{p=p^{(k)}} \quad (4)$$

Thus we have:

$$p^{(k+1)} = p^{(k)} + \Delta p^{(k)} = p^{(k)} + (A^{(k)T} \cdot A^{(k)})^{-1} \cdot A^{(k)T} \cdot \Delta d^{(k)} \quad (5)$$

In practice, we employ an alternating approach by the following procedure.

(i) Suppose that the target is at the reference position. Estimate tracker positions.

In x-y-z coordinate system based on the coordinate system of the machining center, the reference target position is given by  $x_i = [x_i \ y_i \ z_i]^T$  ( $i=1,2,\dots,m$ ). In this coordinate system, the unknown tracker position is given by  $X_j = [X_j \ Y_j \ Z_j]^T$  ( $j=1,2,3,4$ ). The function,  $g_{ij}$ , relating  $X_j$  and  $d_{ij}$ , is given by:

$$d_{ij} = |x_i - X_j| - |x_1 - X_j| \equiv g_{ij}$$

The relation between  $d$  and tracker positions can be written in the following equation:

$$d - g(X_j) = 0 \quad (6)$$

Tracker positions,  $X_1, X_2, X_3, X_4$ , can be identified by solving Eq.(6) in an analogous iterative manner as in Eq.(5)

(ii) Suppose that tracker positions are given as in (i). Estimate target positions.

In u-v-w coordinate system (see to Fig. 4), tracker positions estimated in (i) is converted into  $\hat{U}_j$ . The unknown target position is given by  $u_i = [u_i \ v_i \ w_i]^T$  ( $i=1,2,\dots,m$ ). The function,  $h_{ij}$ , relating  $U_j$  and  $d_{ij}$ , is given by:

$$d_{ij} = |u_i - \hat{U}_j| - |u_1 - \hat{U}_j| \equiv h_{ij}$$

The relation between  $d$  and  $h$  can be written in the following equation:

$$d - h(u_i) = 0 \quad (7)$$

Target positions,  $u_1 \sim u_m$ , can be identified by solving Eq.(7) in an analogous iterative manner as in Eq.(5)

(iii) Repeat (i)-(ii) until target positions and tracker positions become converge.

### 3. Experiments

#### 3.1 Experimental Setup

Experiments to evaluate the performance of the laser tracker used in our study were conducted. The volumetric accuracy of a vertical machining center of major specifications shown in Table 1 was measured by using the laser tracker.

Table 1 Major specification of the machining center

|            |                         |
|------------|-------------------------|
| Travel     | X:610mm Y:510mm Z:460mm |
| Drive type | Ball screw              |
| Guideway   | Roller                  |
| Resolution | 1 $\mu$ m(X, Y, Z)      |

A corner cube is attached to the machine's spindle as the target. It is located at total 40 points shown in Fig. 5. Nominal target positions are within 100mm  $\times$  100mm  $\times$  100mm. Since only one laser tracker is available at this stage, it is set in different four positions on the machine table (A ~ D in Fig. 5), and laser displacements from each tracker location to total 40 target locations above are measured. Figure 6 shows a photograph of experimental setup.

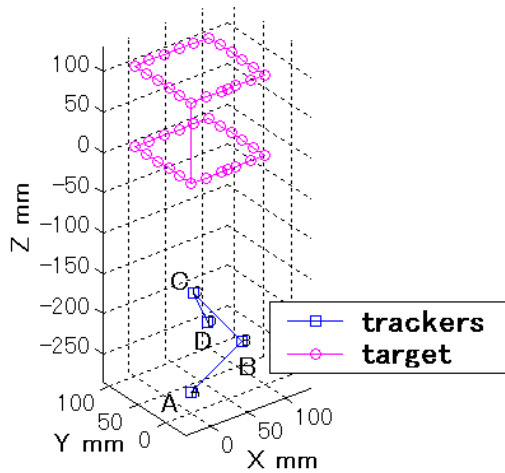


Fig. 5 Location of trackers and measurement range

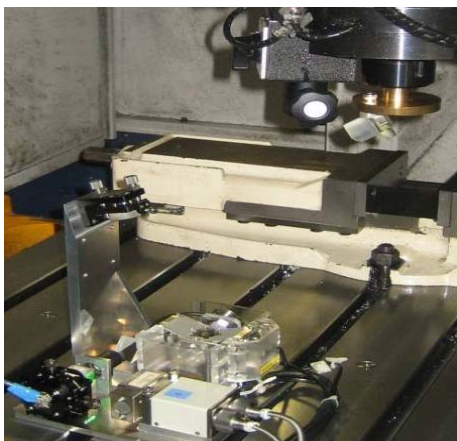


Fig. 6 An outlook of experiment

#### 3.2 Experimental Results

Measured laser displacements from each tracker and the target are shown in Fig. 7.

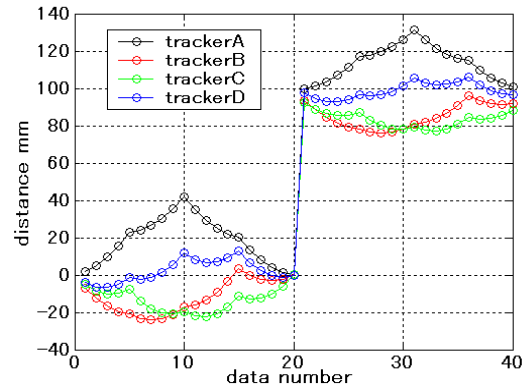


Fig. 7 Laser displacement

For each reference target location, the actual target location is estimated from laser displacements by applying the algorithm presented in Section 2.2. Figure 8 shows error vectors of the estimated target position from its reference position. Figure 9 shows its projection on the XY plane. The error is magnified by 100 times. In the same way, Fig. 10 shows its projection on the XZ plane. The error is magnified by 100 times.

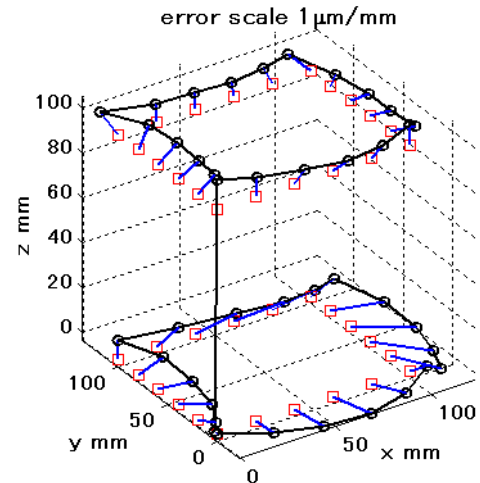


Fig. 8 Volumetric errors

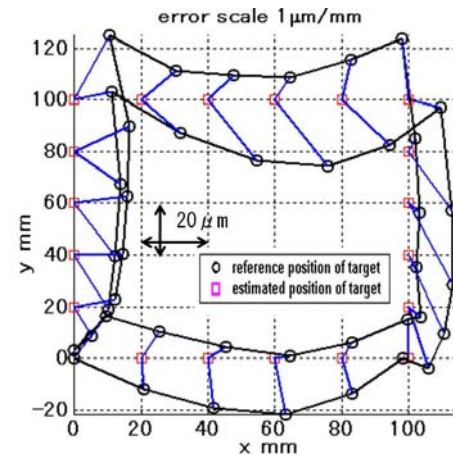


Fig. 9 Estimation errors in the XY plane

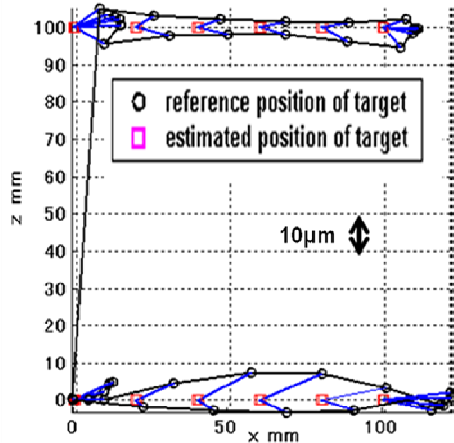


Fig. 10 Estimation errors in the XZ plane

Although the target's actual position was not measured by other more reliable measurement device for the comparison, volumetric errors of the measured machine tool in the region  $100\text{mm} \times 100\text{mm} \times 100\text{mm}$  must be at most several micrometers. It can be said that errors shown in Fig. 8 are mostly caused by estimation errors by the laser tracker.

## 4. Simulations

### 4.1 Condition of Simulation

In Fig. 9 and Fig. 10, estimation errors in the Z-direction are clearly smaller than those in X- and Y-directions. This suggests that four positions of laser tracker affect the estimation accuracy. To find out better setups of laser tracker to improve its estimation accuracy, sensitivity analysis of laser measurement uncertainties to the estimation accuracy of target positions will be presented in this section.

In this simulation, target and tracker positions are shown in Fig. 11. In order to more clearly see the influence of measurement uncertainties, the simplified algorithm to compute target positions is used in the simulation; at Step (ii) in Section 2.2, each target position is calculated from laser displacements from three trackers by the triangulation principle. The following two types of uncertainties in the measured laser displacement are included in the simulation.

#### (I) Random error

Laser displacements,  $d_{ij}$ , have the nominally-distributed random errors of the average 0 and standard deviation of  $1\mu\text{m}$ .

#### (II) Linear error

Laser displacements,  $d_{ij}$ , have an error proportional to the displacement ( $3\mu\text{m}/100\text{mm}$ ).

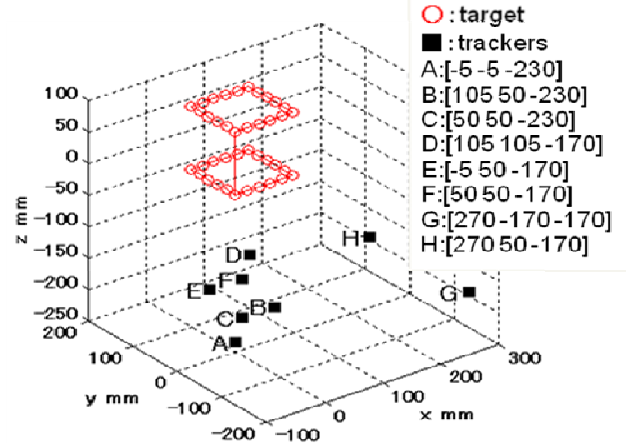


Fig. 11 Location of targets and laser trackers in the simulation

### 4.2 Results of Simulation

As shown in Table 2, the error sensitivity analysis is conducted under six different combinations of tracker positions. It also shows mean and standard deviation of the norm of difference between the estimated target positions and their reference positions.

Table 2 mean and standard deviation of the norm of difference between the estimated target positions and their reference positions.

|             | Location of the laser trackers | Mean and standard deviation of the norm of errors (mm) |                    |
|-------------|--------------------------------|--|--------------------|
|             |                                | Random errors (I)                                      | Linear errors (II) |
| Condition 1 | A , B , C                      | 0.0145±0.0081  | 0.0047±0.0029      |
| Condition 2 | A , B , G                      | 0.0035±0.0023  | 0.0031±0.0015      |
| Condition 3 | A , G , H                      | 0.0027±0.0014  | 0.0024±0.00086     |
| Condition 4 | D , E , F                      | 0.0130±0.0063  | 0.0041±0.0017      |
| Condition 5 | D , E , G                      | 0.0043±0.0022  | 0.0026±0.0010      |
| Condition 6 | E , G , H                      | 0.0033±0.0014  | 0.0024±0.00094     |

From the result of this simulation, the estimation uncertainty by the laser tracking system seems to be dependent on the location of laser trackers relative to target locations. Compared to Condition 1, Condition 3 has significantly smaller estimation errors. It was also verified that estimation errors in X- and Y-directions are as small as those in Z-directions. It is clearly observed that tracker positions G and H in Fig. 11 reduces the sensitivity to measurement errors by comparing condition 1~ 3 and condition 4 ~ 6, however, the improvement of estimation accuracy is not observed. Thus, it can be said that changing the location of laser trackers relative to target locations in the Z-direction only is not effective.

Further study will be conducted to make sure the result of this simulation in terms of experiments. However, setting the laser tracker at G and H positions in Fig. 11 requires wide viewing angle of a target. Thus, instead of a corner cube, we will use a cat's-eye as a target retroreflector [6].

## 5. Conclusions

A laser tracking system can measure the position of a target in the three-dimensional space based on the triangulation principle. It can be potentially applied to the direct measurement of three-dimensional volumetric errors of a machine tool over a large workspace.

In this experiment, the measurable range was  $100\text{mm} \times 100\text{mm} \times 100\text{mm}$ . However, the estimation accuracy of a corner cube position, attached on a spindle of a machining center, was not sufficiently high compared to the positioning error of the measured machine tool.

The result of this experiment and simulation showed that the estimation uncertainty by the laser tracking system is dependent on the location of laser trackers relative to target locations. Error sensitivity analysis showed that wider distribution of tracker positions in XY will improve the estimation accuracy.

We will continue our study to improve its estimation accuracy such that it can be applied to the error calibration of a machining center.

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## References

- [1] Schwenke H, Knapp W, Haitjema H, Weckenmann A, Schmitt R, Delbressine F. "Geometric Error Measurement and Compensation of Machines". *Annals of the CIRP* 57/2:660–675, 2008.
- [2] H. Schwenke, R. Schmitt, P. Jatzkowski, C. Warmann. "On-the-fly calibration of linear and rotary axes of machine tools and CMMs using a tracking interferometer". *Annals of the CIRP* 58/477-480, 2009.
- [3] Umetsu, K. Furutani, R., Osawa, S., Takatsuji, T., Kurosawa, T. "Geometric calibration of a coordinate measuring machine using a laser tracking system," *Meas. Sci. Technol.* Vol. 16, pp. 2466–2472, 2005.
- [4] Soichi Ibaraki, Takafumi Hata, Tomoaki Yano, Toshiyuki Takatsuji, Sonko Osawa, Osamu Sato. "Estimation of Three-dimensional Volumetric Errors of Machine Tools by a Laser Tracker". *ASPEN* 2009.
- [5] Yano, T., Takatsuji, T., Osawa, T., Suzuki, T., Motomura, Y., Itabe, T., "Development of a Small Two Axis Spherical Motor Type Laser Tracker with Submicron Measurement Accuracy," *IEEJ Trans. on Sensors and Micromachines*, Vol. 126, No. 4, pp.144-149, 2006.
- [6] Toshiyuki Takatsuji, Mitsuo Goto, Sonko Osawa, Ruimin Yin, Tomizo Kurosawa, "Whole-viewing-angle cat's-eye retroreflector as a target of laser trackers," *Meas. Sci. Technol.* 10 (1999) N87–N90.