Measurement and compensation of motion errors on 5-axis machine tool by R-test

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Abstract:

5-axis machine tool intended in this study has three linear axes and additional two rotary axes. Because of its composition, errors of each axis and assembly errors are integrated and they influence the relative position between the tool and the workpiece. Efficiently investigation method of motion errors is not conformed enough so far. By experiment, it is sure that errors are varying as the work table rotates. The objective of this paper is how to evaluate the position-dependent geometric errors of 5-axis machine tool and make error map with the use of the measurement device, called "R-test".[

Keywords: R-test, 5-axis machine tool, Measurement, Position-dependent geometric errors.

1. Introduction

A 5-axis machine tool has three linear axes to translate the tool and/or the workpiece and additional two rotary axes to tilt or rotate it. Due to its capability to machine more complex workpieces, it has been popularized rapidly in the manufacturing field. Since a 5-axis machining center has linear and rotary axes that are stacked over each other, motion errors of each axis and its assembly error are accumulated in the positioning error of a tool relative to a workpiece. Consequently, many machine tool users recognize that the machining accuracy with 5-axis machine tool is lower than conventional 3-axis machine tool. For improving the motion accuracy of 5-axis machine tool, it is first crucial to develop a methodology to identify causes of the machine's motion error in an accurate and efficient manner.

 Static position and orientation errors of the axis average line of rotary axes are among the most fundamental error factors of the 5-axis kinematics, and called location errors in ISO 230-7[1] or geometric errors[2]. Many researches have been reported on the identification of geometric errors. Typical one is the measurement methodology with the use of the double ball bar (DBB) [3]. This methodology is currently under the discussion at ISO TC39/SC2[4].

In the measurement with DBB device, one ball is attached to the spindle and the other one is on the work table, and the distance between them is measured with a linear encoder in the bar connecting two balls. Since only one direction displacement is acquired in this measurement, the operator has to change the setup of the measurement device and machine tool at least a couple of times to identify all the geometric errors. Further more, it is difficult for the measurement with the ball bar to measure more complex error factors than geometric errors. For example, the positioning error of rotary axes, the runout of rotary axes, the influence of the gravity are not included in geometric errors, and cannot be identified by ball bar measurement presented in [3]. To perform fully-automated calibration of a larger class of error motions of 5-axis machines, ball bar measurements[3]

have inherent critical issue.

 In this study, we utilize the measurement device called R-test[5][6]. It is commercially available by IBS Precision Engineering, Fidia. The standardization of R-test has been also discussed for ISO 10791-6. R-test can measure the position of the tool in relative to the workpiece in three dimensions, which allows us to obtain more data by one measurement setup than in the ball bar measurement. The objective of this paper is to develop a methodology to identify position-dependent geometric errors, or "error map", of rotary axes in 5-axis machine tools by using the R-tests.

2. Configuration of 5-axis Machine Tool and Error Parameters

2.1 Configuration of 5-axis Machine Tool

The configuration of 5-axis machine tool considered in this paper is shown in Fig. 1. This machine tool has two rotary axes, B and C-axis, to tilt the work table. When the C- and Z-axes are parallel to each other as shown in Fig. 1, the B-axis is defined as $B=0^{\circ}$

Figure 1: Configuration of 5-axis machine tool

2.2 Position-dependent Geometric Errors

As shown in Table 1, six position and orientation error

parameters are defined on each rotary axis. These parameters for the axis average line of each rotary axis are defined in ISO230-7.

Practically, these error parameters may vary as the rotary axis rotates. In this paper, these are considered as a function of the angular position of rotary axis, and referred to as position-dependent geometric errors. Position-dependent geometric errors can model more complex error motions. The errors on B-axis are defined as a function of the angular position of B-axis itself. The errors on C-axis are defined as a function of the B- and C-axes (e.g. $\delta x_{BY}(B_i)$, $\delta x_{CB}(B_i, C_i)$).

 The errors on linear axes are ignored in this study, assuming that they are sufficiently small in comparison with errors on rotary axes.

2.3 Kinematic Modeling of 5-axis Machine Tool

 The kinematic model to compute the relative position of the tool to the workpiece is the basis of modeling the influence of motion errors on rotary axes to R-test measurements. This subsection presents the model briefly. Define the machine coordinate system as the coordinate system fixed on the machine frame, and the workpiece coordinate system as the coordinate system fixed on the work table.

 Homogeneous transformation matrix (HTM) to represent the translation to X, Y, and Z-direction for the distance of *X*, *Y*, and *Z*, and rotation about X, Y, and Z-axis for the angle of *A*, *B*, and *C*, are shown as follows:

$$
D_x(X) = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_a(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & -\sin A & 0 \\ 0 & \sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_y(Y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_b(B) = \begin{bmatrix} \cos B & 0 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ -\sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_z(Z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_c(C) = \begin{bmatrix} \cos C & -\sin C & 0 & 0 \\ \sin C & \cos C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
 (1)

 In this paper, the commanded tool center point in the workpiece and machine coordinate systems is respectively described as $^w p^*$ and $^r p^*$. Its actual position influenced by the motion errors are described as $\int_{a}^{w} p$ and $\int_{a}^{r} p$. The left-side subscript *r* and *w* respectively represents a vector in the machine and workpiece coordinate systems. The right-side subscript * represents the commanded position.

 When the commanded tool center point in the workpiece coordinate system at the angular command (*B*, *C*) is given by ${}^w p^* = [{}^w p_x^*, {}^w p_y^*, {}^w p_z^*]^T$, it can be transformed in the machine coordinate system by the HTM as follows:

$$
\begin{bmatrix} {^r p^*} \\ 1 \end{bmatrix}^T = {^r \widetilde{T}_w} \begin{bmatrix} {^w p^*} \\ 1 \end{bmatrix}^T
$$
 (2)

where ${}^{r} \widetilde{T}_{w}$ is the HTM that can transform from the tool center point in the workpiece coordinate system into the machine coordinate system with angular commands:

$$
{}^{r}\widetilde{T}_{w} = D_{b}(B)D_{c}(C) \tag{3}
$$

Note that the displacement sensors are fixed on the rotary table and tool center point is measured in the workpiece coordinate system. The relative position of the tool with respect to the sensors is represented as:

$$
\begin{bmatrix} {}^{w} p \\ 1 \end{bmatrix} = \left({}^{r} T_{w} \right)^{1} {}^{r} \widetilde{T}_{w} \begin{bmatrix} {}^{w} p^{*} \\ 1 \end{bmatrix}
$$
 (4)

where T_w is the HTM with error parameters in Table 1:

$$
{}^{r}T_{w} = {}^{r}T_{B} {}^{B}T_{w} \tag{5}
$$

$$
{}^{r}T_{B} = D_{x}(\delta x_{BY})D_{y}(\delta y_{BY})D_{z}(\delta z_{BY})
$$

$$
D_{a}(\alpha_{BY})D_{b}(\beta_{BY})D_{c}(\gamma_{BY})D_{b}(B)
$$
 (6)

$$
{}^{B}T_{w} = D_{x}(\delta x_{CB})D_{y}(\delta y_{CB})D_{z}(\delta z_{CB})
$$

$$
D_{a}(\alpha_{CB})D_{b}(\beta_{CB})D_{c}(\gamma_{CB})D_{c}(C)
$$
 (7)

 $D_a(\alpha_{CB})D_b(\beta_{CB})D_c(\gamma_{CB})D_c(C)$
Denoting $^w p=[^w p_x, ^w p_y, ^w p_z]^T$, Eq. (4) can be rewritten as:

$$
\begin{bmatrix} {}^{w}P_{x} {}^{w} \\ {}^{w}P_{y} {}^{w} \\ {}^{w}P_{z} {}^{w} \end{bmatrix} = \begin{bmatrix} 1 & -\Delta C & \Delta B & \Delta X \\ \Delta C & 1 & -\Delta A & \Delta Y \\ -\Delta B & \Delta A & 1 & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{w}P_{x} {}^{*} \\ {}^{w}P_{y} {}^{*} \\ {}^{w}P_{z} {}^{*} \end{bmatrix}
$$
(8)

When each error parameter is sufficiently small, we have: $\Delta X = {\delta y_{BY}(B) + \delta y_{CB}(B, C)} \sin C$

+{
$$
\delta \chi_{BY}(B)
$$
cos B- $\delta \chi_{BY}(B)$ sin B+ $\delta \chi_{CB}(B, C)$ }cos C
\n
$$
\Delta Y = {\delta \gamma_{BY}(B) + \delta \gamma_{CB}(B, C)}
$$
cos C
\n-{ $\delta \chi_{BY}(B)$ cos B- $\delta \chi_{BY}(B)$ sin B+ $\delta \chi_{CB}(B, C)$ }sin C
\n
$$
\Delta Z = \delta \chi_{BY}(B)
$$
sin B+ $\delta \chi_{BY}(B)$ cos B+ $\delta \chi_{CB}(B, C)$
\n
$$
\Delta A = {\beta_{BY}(B) + \beta_{CB}(B, C)}
$$
sin C
\n+{ $\alpha_{BY}(B)$ cos B- $\gamma_{BY}(B)$ sin B+ $\alpha_{CB}(B, C)$ }cos C
\n
$$
\Delta B = {\beta_{BY}(B) + \beta_{CB}(B, C)}
$$
cos C
\n
$$
-\{\alpha_{BY}(B) \cos B - \gamma_{BY}(B) \sin B + \alpha_{CB}(B, C)
$$
}sin C
\n
$$
\Delta C = \alpha_{BY}(B) \sin B + \gamma_{BY}(B) \cos B + \gamma_{CB}(B, C)
$$

3. Measurement with R-test device

3.1 Outlines of R-test Device

The overview of R-test device is shown in Fig. 2. This device is composed of the ball attached on the machine spindle and three displacement sensors attached on the rotary table. Sensors are fixed to direct approximately to the center of the ball. By displacements measured by three sensors, the relative deviation of the ball from the reference position can be computed.

Figure 2: Overview of R-test device

3.2 Calibration of the Device

To get ball position from displacements of three sensors, the direction vectors of each sensor need to be identified in advance. Furthermore, since the center shift of the ball from the spindle centerline significantly influences measurement results, it must be calibrated in prior and compensated.

First, give a set of certain known command values to linear axes and move the ball. Assuming that the positioning error of linear axes is negligibly small, the direction vector of each sensor can be identified from a set of measured ball displacements.

Then, by indexing the spindle at a set of given command angular positions, get the displacements of the sensors at each rotation angle and identify the center shift of the ball.

3.3 Measurement procedure

In the measurement with R-test device, the displacement of each sensor is acquired at various angular positions of rotary axes. In this study, B-axis is set at every 30° from -90° to 90° , and C-axis is set at every 30° from 0° to 330°, to cover the entire movable range of rotary axes. The number of measurement points is 84. The linear axes are commanded such that the ball follows the sensors on the rotary table. When reading out the values of the sensors, all of the axes are rested.

To observe the orientation of C-axis of rotation, at least two sets of R-test measurements must be done with different sensor locations. More details will be give in Section 4.2.

4. Identification of Position-dependent geometric errors

4.1 Identification of errors on B-axis

In this subsection, the algorithm for identifying the position-dependent geometric errors of B-axis is presented.

 First, it is assumed that all position-dependent geometric errors of C-axis are zero. Errors of B-axis at $B=B_i(i=1 \sim 7)$ are described as the vector:

$$
w(B_i) = \left[\delta x_{BY}(B_i), \delta y_{BY}(B_i), \alpha_{BY}(B_i), \beta_{BY}(B_i), \gamma_{BY}(B_i)\right]^T
$$
\n(10)

The position of the ball in the workpiece coordinate system at $B = B_i$ and $C = C_j$ is given by:

$$
{}^{w}p(B_{i},C_{j}) = \left[{}^{w}p_{x}(B_{i},C_{j}) - {}^{w}p_{y}(B_{i},C_{j}) - {}^{w}p_{z}(B_{i},C_{j}) \right]^{r}
$$
 (11)

The Jacobian matrix representing the relation between $w(B_i)$ and $^w p(B_i, C_j)$ is given by:

$$
\frac{\partial^w p(B_i, C_j)}{\partial w(B_i)} = \begin{bmatrix} \frac{\partial^w p_x(B_i, C_j)}{\partial \hat{\alpha}_{\mathbf{F}_i}(B_i)} & \cdots \\ \frac{\partial^w p_y(B_i, C_j)}{\partial \hat{\alpha}_{\mathbf{F}_i}(B_i)} & \cdots \\ \frac{\partial^w p_y(B_i, C_j)}{\partial \hat{\alpha}_{\mathbf{F}_i}(B_i)} & \cdots \\ \frac{\partial^w p_z(B_i, C_j)}{\partial \hat{\alpha}_{\mathbf{F}_i}(B_i)} & \cdots \end{bmatrix}
$$
(12)

When all components of $w(B_i)$ are sufficiently small, the following linear approximation is holds:

^w
$$
p(B_i, C_j) \approx \frac{\partial^w p(B_i, C_j)}{\partial w(B_i)} \cdot w(B_i)
$$
 (13)

As described in the previous section, in R-test measurement, only the relative displacement of the ball from its original position can be measured. In this paper, for the simplicity of notation, $B_1 = C_1 = 0^\circ$ is defined as the original position. In other words, $^w p(B_i, C_j)$ is always [0 0 0]^T at $B_1 = C_1 = 0^\circ$. For this reason,

$$
{}^{w} p(B_i, C_j) =
$$

\n
$$
\frac{\partial^{w} p(B_i, C_j)}{\partial w(B_i)} \cdot w(B_i) - \frac{\partial^{w} p(B_i, C_1)}{\partial w(B_1)} \cdot w(B_1)
$$
\n(14)

A set of position-dependent geometric errors, $w(B_i)$ can be obtained by solving the following optimization problem by using the least square method:

$$
\min_{w} \sum_{j} \left\| w p(B_i, C_j) - \left(\frac{\partial^w p(B_i, C_j)}{\partial w(B_i)} w(B_i) - \frac{\partial^w p(B_i, C_1)}{\partial w(B_i)} w(B_i) \right) \right\|^2 \tag{15}
$$

Note that geometric errors at $B_1=0^\circ$, $w(B_1)$, must be first identified and then substituted in Eq. (15) for other *Bi's.*

 Equation (8) represents the relationship between the commanded position ψ_p^* and measured position ψ_p in workpiece coordinate system. By differentiating partially with each error parameter, the Jacobian matrix in Eq. (12) is given.

4.2 Identification of errors on C-axis

 Define the coordinate system that is fixed on the B-axis and rotates concurrently with B-axis rotation as *B-axis coordinate system*. In this coordinate system, the commanded position of the ball is given by ${}^B p^*(B_i, C_j)$ (Eq. (16)) and its measured position is given by ${}^B p(B_i, C_i)$ (Eq. (17)).

$$
\begin{bmatrix} {^B}p^*(B_i, C_j) \\ 1 \end{bmatrix} = D_c(C_j) \begin{bmatrix} {^w}p^*(B_i, C_j) \\ 1 \end{bmatrix}
$$
 (16)

$$
\begin{bmatrix} {^B}p(B_i, C_j) \\ 1 \end{bmatrix} = D_c(C_j) \begin{bmatrix} {^w}p(B_i, C_j) \\ 1 \end{bmatrix}
$$
 (17)

Identify the position-dependent geometric errors on C-axis by the following procedure.

 First, compute the influence of the position-dependent geometric errors of B-axis identified in the previous subsection in the B-axis coordinate system as:

$$
\begin{bmatrix} {^B \hat{p}(B_i, C_j)} \\ 1 \end{bmatrix} = {^B T_w} \begin{bmatrix} {^w p^*(B_i, C_j)} \\ 1 \end{bmatrix}
$$
 (18)

where ${}^{B}T_{W}$ is the HTM to transform the ball position in the workpiece coordinate system into the B-axis coordinate system given by:

$$
{}^{B}T_{w} = D_{b}(-B_{i})D_{x}(\delta x_{BY}(B_{i}))\cdots D_{c}(\gamma_{BY}(B_{i}))D_{b}(B_{i})D_{c}(C_{j})
$$
\n(19)

Subtract $^B \hat{p}(B_i, C_i)$ from the measured position by:

$$
{}^{B}q(B_i, C_j) = {}^{B}p^*(B_i, C_j) + {}^{B}p(B_i, C_j) - {}^{B}\hat{p}(B_i, C_j)
$$
 (20)

This represents the ball position that is influenced by only C-axis errors.

This process is conducted twice with the displacement sensor located at different positions on the rotary table. As shown in Fig. 3, the orientation of the line connecting ${}^B q_i(B_i, C_j)$ and ${}^{\bar{B}}q_2(B_i, C_j)$ with respect to their command positions represents the orientation error of C-axis. By projecting this to Y-Z plane, $\alpha_{CB}(B_i, C_j)$ can be acquired. In the same way, by projecting this to X-Z plane, $\beta_{CB}(B_i, C_i)$ can be acquired. In this study, we assume that the angular positioning error of C-axis is sufficiently small, i.e. $\gamma_{CR}(B_i, C_i)=0$.

 Then, identify the translational errors on C-axis. By using acquired rotational errors of C-axis, the influence of these errors to the ball position in B-axis coordinate system is simulated as following:

$$
\begin{bmatrix} {^{B}q}(B_{i},C_{j}) \ 1 \end{bmatrix} = D_{a}(\alpha_{CB}(B_{i},C_{j}))D_{b}(\beta_{CB}(B_{i},C_{j}))D_{c}(\gamma_{CB}(B_{i},C_{j})) \begin{bmatrix} {^{B}p}^{*}(B_{i},C_{j}) \ 1 \end{bmatrix}
$$
\n(21)

Therefore, translational errors of C-axis are given by:

$$
\begin{vmatrix}\n\delta_{CB}(B_i, C_j) \\
\delta_{CB}(B_i, C_j) \\
\delta_{CB}(B_i, C_j)\n\end{vmatrix} = ^B q(B_i, C_j)^{-B} q'(B_i, C_j)
$$
\n
$$
B_{p_2^*(B_i, C_j)} \longrightarrow \begin{pmatrix}\n\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j)\n\end{pmatrix}
$$
\n
$$
B_{p_1^*(B_i, C_j)} \longrightarrow \begin{pmatrix}\n\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j)\n\end{pmatrix}
$$
\n
$$
B_{p_1^*(B_i, C_j)} \longrightarrow \begin{pmatrix}\n\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j) \\
\alpha_{CB}(B_i, C_j)\n\end{pmatrix}
$$

Figure 3: Computation of orientation errors of C-axis, for example $\alpha_{CB}(B_i, C_i)$, by means of two measurements. The right-side subscript 1 donets the first measurement, 2 donates second measurement. The measured position of the ball is represented by ●, the simulated position is represented by ○.

5. Experimental Case Study

5.1 Measurement

Commanded positions of each axis are shown in Fig. 4.

As described in Section 3.3, a set of R-test measurements was performed on the experimental machine of the configuration shown in Fig. 1.

Then, compute the ball positions by acquired sensor displacements. Since it is assumed that the motion errors on linear axes are sufficiently small in this study, the ball displacement measured by R-test, $^w p(B_i, C_j)$, can be seen as the table displacement by taking $\cdot \ ^{w}p(B_i,C_j)$. Table displacements measured by the R-test procedure are shown in Fig. 5. The positions of sensor projected on X-Z plane at B=0° are shown in (a), and projected on the table surface at $B=0^{\circ}$ are shown in (b). In the same way, the positions of the sensor at B=90° are shown in (c) and (d). Commanded positions of the sensor are represented by ●, measured position is represented by ○. The deviation between commanded and measured positions is magnified by a factor of 10,000.

Figure 5: Commanded and measured table positions by R-test measurement (Errors are magnified by a factor of 10,000).

5.2 Identification of Position-dependent Geomeric Errors and Simulation

Position-dependent geometric errors are identified from acquired position errors by using the algorithm presented in Section 4. In Fig. 6, identified rotation errors of B-axis are represented in (a), translational errors of B-axis are in (b). Position-dependent geometric errors of C-axis at $B=0^{\circ}$ and $B=90^{\circ}$ are represented in Fig. 7.

 To illustrate the influence of B- and C-axes error motions, the influence of identified position-dependent geometric errors of B-axis only, shown in Fig. 6, is computed by Eq. (18) and:

$$
\begin{bmatrix} r & p \\ 1 & 1 \end{bmatrix} = r_{w} \begin{bmatrix} w & p^* \\ 1 & 1 \end{bmatrix}
$$
 (23)

where ${}^{r}T_{w}$ is given by Eq. (5).

 The result is shown in Fig. 8. Commanded positions of the sensor are represented by ●, measured positions are represented by ○, and simulated positions are represented by \times .

Figure 6: Identified position-dependent geometric errors on B-axis

5.3 Observation

The following observations can be made:

- Identified $\delta x_{BY}(B_i)$ and $\delta y_{BY}(B_i)$ (Fig. 6(b)) have an error about 4 μm and 6 μm, respectively, in average. This is mostly caused by the mis-calibration of C-axis centerline in X and Y directions.
- The table position at $B=90^\circ$ is shifted by about 10 μ m to *–*Z direction.

Figure 7: Identified position-dependent geometric

- Identified $\delta z_{BY}(B_i)$ is about 13 μm at B=0°, and get about 19 μm at B=–90°. $\delta z_{BY}(B_i)$ at B=0° indicates the mis-calibration of B-axis centerline in Z-direction. If it is merely the mis-calibration, $\delta z_{BY}(B_i)$ must be the same for any B angular positions. The increase in $\delta z_{BY}(B_i)$ at B=90° suggests that the table is shifted in –Z direction, likely due to the gravity influence.
- Identified $\alpha_{BY}(B_i)$ (Fig. 6(a)) varies from -13×10^{-3} ^o at B=–90° to $+13\times10^{-3}$ ° at B=90°. Identified $\gamma_{BY}(B_i)$ also

shows analogous variation. This suggests that the **6. Concluding Remark** B-axis has a "coning" error motion. The position-dependent geometric errors are identified

- Almost no angular position error of B-axis is observed (Fig. 6(a), β*BY*(*Bi*)).
- $\alpha_{CB}(B_i, C)$ and $\gamma_{CB}(B_i, C_i)$ (Fig. 7) show a sinusoidal error motions. This suggests that the C-axis also shows a "coning" error motion. The amplitude is similar at $B=0^\circ$ and $B=90^\circ$.

Figure 8: Ball positions commanded, measured by R-test measurement, and simulated (Errors are magnified by a factor of 10,000)

efficiently and automatically with the use of R-test device. When the error map can be made with identified positiondependent geometric errors, the motion accuracy can be improved by compensating them.

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