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Taming Internal Dynamics by Mismatched and H_∞-Optimized State Observer

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Abstract

The feedback linearization control scheme is simple but quite effective for control of a single-input single-output (SISO) linear parameter-varying (LPV) plant. When the relative degree of the plant is less than its order, the feedback linearization control scheme generates the internal dynamics, which may or may not be stable. This paper presents the design methodology of a state observer that not only provides the feedback linearization controller with good estimation of state variables of the plant, but also stabilizes the overall closed-loop system. We propose to make the choice of system matrices of the state observer completely open in order to search for such an observer structure. Two application examples, including application to steering control for lateral motion of heavy-duty vehicles, are presented to illustrate how the proposed mismatched observer stabilizes the closed-loop system.

1 Introduction

Linear parameter-varying (LPV) systems are linear time-varying plants whose state-space matrices are fixed functions of a vector of known time-varying parameters, $\theta(t)$. They can be generally described by state-space equations of the following form:

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) \\ y(t) &= C(\theta(t))x(t) + D(\theta(t))u(t) \end{aligned} \tag{1}$$

The simplest gain-scheduled controller design methodology for this class of plants is linear interpolation of locally designed linear controllers. This approach is effective only under the assumption that the scheduling parameter $\theta(t)$ changes slowly [1]. Researchers in the H_∞ robust control field have developed the linearly-interpolated controller that guarantees the stability and H_∞ norm performances of the closed-loop system for any trajectory of the parameter $\theta(t)$ [2].

Those linear interpolation methods essentially "approximate" a nonlinear plant by linear combination of a multiple of local linear models. On the other hand, the feedback linearization control (e.g. [5]) cancels all parameter-dependent terms in the closed-loop input-output dynamics without using any linear approximation. It can be seen as a gain-scheduled state feedback controller in the sense that its state feedback gain matrix is dependent on the scheduling parameters.

When the relative degree of the plant is less than its order, the internal dynamics arises in the closed-loop system under the feedback linearization control, which may or may not be stable. This occurs when the zero dynamics of the system is unstable. This paper considers the stabilization of systems with unstable internal dynamics. Conventional methods for stabilization of the internal dynamics include employment of the "outer-loop" linear feedback controller (e.g. [3]). Employment of the "outer-loop" controller makes, however, the entire controller structure much more complicated. Furthermore, the closed-loop performance is likely dominated by the outer-loop controller, not by the inner-loop feedback linearization controller, which sometimes obscures the role of the feedback linearization controller.

If the feedback linearization control law includes the state feedback terms and the state vector is not directly available, it is necessary to design a state observer. This paper presents the design methodology of a state observer that not only provides the feedback linearization controller with good estimation of state variables of the plant, but also stabilizes the overall closed-loop system. When the classical Luenberger-type state observer is used, the internal dynamics resulting from the state feedback control law is preserved in the observer state feedback system (the separation theorem for the closed-loop eigenvalues for observer state feedback control [4]). While it may be argued that the feedback linearization scheme should not be used if the zero dynamics is unstable, the use of mismatched observer avoids to make it internal dynamics.

We make the choice of state matrices of the observer completely open such that we can search for the structure that works on stabilization of the closed-loop system, as well as state estimation. In the Luenberger observer, only the observer gain matrix can be tuned to obtain the desired estimation error dynamics. The LMI-based H_∞ optimization algorithm can be applied to optimize all of system matrices of the state observer such that it stabilizes the overall closed-loop system. By cooperating with the H_∞ loop-shaping technique, the set of system matrices of the observer can be optimized such that it outputs the estimates of the plant state variables with the desired estimation error dynamics.

The remainder of this paper is organized as follows. In the next section, the feedback linearization control scheme is outlined. The proposed design methodology of the state observer is presented in Section 3. Section 4 presents two application examples. In the first example, the feedback linearization controller and the proposed mismatched observer were applied to a second order system to illustrate how the proposed mismatched observer stabilizes

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the entire closed-loop system. The second example is the steering control problem for lateral control of heavy-duty vehicles (HDVs). Numerical simulations are conducted to show time-domain closed-loop performances of the designed controller and observer.

2 Feedback Linearization Control Scheme

The feedback linearization control scheme determines the control input such that it cancels all nonlinear terms in the closed-loop input-output dynamics. See e.g. [5] for its general formulation. Here, as an example, consider a SISO second-order dynamic system of the following form:

$$\begin{aligned} \ddot{q}(t) + A_{22}(\theta)\dot{q}(t) + A_{21}(\theta)q(t) &= B(\theta)u(t) \\ y(t) &= C(\theta)q(t) \end{aligned} \quad (2)$$

where $q(t)$ is a vector of n state variables, $u(t)$ is a scalar control input, $y(t)$ is a scalar output and θ is a vector of scheduling parameters. Suppose $C(\theta)B(\theta) \neq 0$ for any θ. Define the control law by

$$u(t) = \frac{1}{C(\theta)B(\theta)}\{C(\theta)A_{21}(\theta)q(t) + C(\theta)A_{22}(\theta)\dot{q}(t) - k_1\dot{y}(t) - k_2y(t) - k_3\int_0^t y(\tau)d\tau + v(t)\} \quad (3)$$

where k_1, k_2 and k_3 are constant. The closed-loop input-output dynamics becomes

$$\ddot{y}(t) + k_1\dot{y}(t) + k_2y(t) + k_3\int_0^t y(\tau)d\tau = v(t) \quad (4)$$

The integral term in the control law (3) is added to reject static disturbances. Notice that Equation (4) is completely independent of θ.
Equation (2) can be rewritten in the standard state space representation by defining the state vector $x = [q(t)^T \; \dot{q}(t)^T]^T$. This can be combined with the control law (3) to describe the closed-loop dynamics. Notice that there are only three tunable controller parameters, while the closed-loop system is $(2n+1)$th order. Therefore, it is impossible to assign all closed-loop poles by the choice of the controller gains. The internal modes that cannot be controlled by the feedback linearization controller are called internal dynamics, which may or may not be stable.

3 Mismatched and H_∞-Optimized State Observer

One way to deal with unstable internal dynamics is to employ an "outer-loop" linear controller [3]. The entire closed-loop configuration is shown in Figure 1. A linear time-invariant (LTI) H_∞ controller is used as an outer-loop controller in order to stabilize the entire closed-loop system and enhance its robustness.
In this paper, we propose to design the state observer such that it not only provides good estimates of the plant state variables, but also stabilizes the entire closed-loop system. The closed-loop configuration is shown in Figure 2. Advantages of the proposed approach are clear; first, the overall controller structure is much simpler. Furthermore,

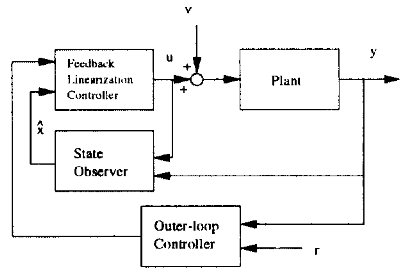

Figure 1: Closed-loop configuration with the outer-loop controller

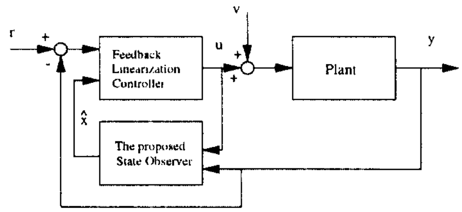

Figure 2: Closed-loop configuration with the proposed mismatched observer

the desired closed-loop input-output dynamics is easier to obtain since, in the outer-loop controller approach, the overall input-output dynamics is often determined by the outer-loop controller, not by the inner-loop feedback linearization controller.
When the classical Luenberger state observer is used for estimating the state vector in the feedback linearization control law, the internal dynamics is completely preserved in the overall closed-loop system. The full-order Luenberger state observer is given by:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - C\hat{x}) \quad (5)$$

where (A, B, C) are system matrices of the plant, $\hat{x}(t)$ is the estimated state vector, and L is the observer gain matrix. Consider the observer state feedback law: $u(t) = F\hat{x}(t) + v(t)$. The separation theorem for the closed-loop eigenvalues for observer state feedback control [4] states that every eigenvalue of the overall closed-loop system is given by either that of $A + BF$ or $A - LC$. That is, the eigenvalues of the combined system of the plant and controller are completely preserved in the overall closed-loop system. Notice that $A + BF$ includes the modes in the internal dynamics. In other words, the Luenberger observer does not affect the internal dynamics.
We propose to make the choice of system matrices of the observer completely open. Consider the state observer of the following structure:

$$\begin{aligned} \dot{x}_o(t) &= A_ox_o(t) + B_{o1}u(t) + B_{o2}y(t) \\ \hat{x}(t) &= C_ox_o(t) + D_{o1}u(t) + D_{o2}y(t) \end{aligned} \quad (6)$$

The state estimate $\hat{x}$ is given as the output vector of the observer, not as the state vector. The observer system matrices $(A_o, B_{o1}, B_{o2}, C_o, D_{o1}, D_{o2})$ are not restricted to coincide with the system matrices of the plant. Even their sizes do not have to coincide with those of the plant system matrices.
The set of those system matrices is required to satisfy the following design objectives: 1) the observer outputs

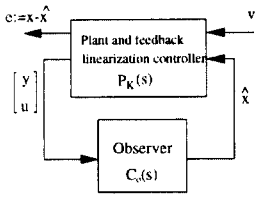

Figure 3: H_∞ synthesis of the state observer

the estimated state variables of the plant with desired estimation error dynamics and 2) the overall closed-loop system is stabilized. They can be written in the form of the H_∞ norm performance objective as follows.

$$\|W_o(s)F_L(P_K(s), C_o(s))\|_\infty < 1 \qquad (7)$$

where $W_o(s)$ is a performance filter transfer function matrix and $F_L(P_K(s), C_o(s))$ denotes the closed-loop dynamics from v to $e := x - \hat{x}$ as shown in Figure 3. $P_K(s)$ denotes the combined system of the plant and the controller and $C_o(s)$ is the state observer dynamics. The standard H_∞ controller synthesis algorithm [6] can be applied to search for such a state observer structure, since the choice of the observer system matrices is completely open.

In the Luenberger observer, only the observer gain matrix L can be tuned to obtain the desired estimation error dynamics. By making every system matrix open for tuning, the observer structure that stabilizes the entire closed-loop system could be found. Notice that this observer is inherently mismatched (i.e. the system matrices of the observer do not match those of the plant) and thus the separation theorem for the closed-loop eigenvalues does not apply. In other words, the closed-loop input-output dynamics is affected by the observer dynamics. By appropriately choosing the performance filter $W_o(s)$ in Equation (7) and designing the estimation error dynamics of the observer properly, however, the effect of the observer dynamics on the closed-loop input-output dynamics may be localized to unstable internal dynamics. Notice that if the observer can estimate the plant state variables perfectly, the closed-loop input-output dynamics would be given by Equation (4) and the unstable internal dynamics will be retained.

Finally, note that the observer design methodology presented in this section assumes that all scheduling parameters are fixed and thus the plant can be regarded as an LTI model. When the scheduling parameters are varying, although the closed-loop input-output dynamics is still time-invariant due to cancellation of parameter-dependent terms by the feedback linearization controller, the internal dynamics becomes time-varying. If variations of scheduling parameters cause serious performance deterioration, more advanced design theories could be applied to observer design, such as the μ-synthesis or the gain-scheduled H_∞ controller design methodology [2].

4 Application Examples

4.1 Application to a Second-order LTI System

Consider a second-order system of the following form:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= a_1x_1(t) + a_2x_2(t) + u(t)\\ y(t) &= x_1(t) - x_2(t)\end{aligned} \qquad (8)$$

where $u(t)$ is a scalar control input and $y(t)$ is a scalar system output. For simplicity, let a_1 and a_2 be constant ($a_1 = a_2 = 1$). The feedback linearization control law is given by

$$u(t) = -a_1x_1(t) - (1 + a_2)x_2(t) - ky(t) + v(t) \qquad (9)$$

where k is constant and $v(t)$ is a new input. Then, the closed-loop input-output dynamics become

$$\dot{y}(t) + ky(t) = v(t) \qquad (10)$$

The overall closed-loop dynamics is given by combination of Equation (8) and (9):

$$\frac{d}{dt}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ k & 1-k\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}v(t) \qquad (11)$$

The closed-loop poles are at $s = 1, -k$. The pole at $s = -k$ dominates the closed-loop input-output dynamics. The unstable pole at $s = 1$ is unobservable from the output and defines the internal dynamics. Note that it cannot be altered by the controller gain. Here, k is set to 0.1.

Notice that the control law (9) assumes that both state variables are available. The Luenberger observer is formulated by

$$\frac{d}{dt}\begin{bmatrix}\hat{x}_1(t)\\ \hat{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ a_1 & a_2\end{bmatrix}\begin{bmatrix}\hat{x}_1(t)\\ \hat{x}_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t) + \begin{bmatrix}l_1\\ l_2\end{bmatrix}e(t) \qquad (12)$$

where $e(t) := y(t) - (\hat{x}_1(t) - \hat{x}_2(t))$. Figure 4 (right) shows the closed-loop pole locations ($l_1 = 10$ and $l_2 = 5$). Compared with the closed-loop poles of Equation (11) shown in the same figure, it can be seen that the Luenberger observer simply adds two new poles. The pole at $s = 1$ is not affected at all by the Luenberger observer and the closed-loop system is unstable.

Next, the proposed mismatched observer is designed for this plant. The design objective is given by

$$\left\|W_o(s)T^{cl}_{v\to x-\hat{x}}(s)\right\|_\infty \leq 1 \qquad (13)$$

where $W_o(s) = \frac{5\times10^{-4}(s+10)}{s+5\times10^{-4}}$ is the performance filter. $T^{cl}_{v\to x-\hat{x}}(s)$ denotes the closed-loop transfer function matrix from $v(t)$ to $x(t) - \hat{x}(t)$. Figure 4 (right) shows the closed-loop poles with the designed mismatched observer. Notice that the unstable internal dynamics has been replaced by a dynamic mode governed by the new stable pole near $s = -2$. The input-output dynamics pole at $s = -k = -0.1$ is still preserved. Two extra poles, which may be interpreted as observer poles*, are introduced near the input-output dynamics pole. These two modes associated with the observer and the mode governed by the stable pole near $s = -2$ are significantly faster than the original input-output dynamics due to the pole at $s = -k = -0.1$. It makes sense to replace the zero dynamics, which does not show up in the input-output response, by dynamics much faster than the input-output dynamics.

Figure 5(a) and (b) show the output response (y) and

*Since the separation principle does not apply, it is not strictly correct to call them observer poles.

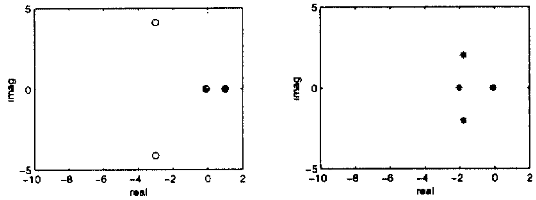

Figure 4: Closed-loop pole locations with direct state feedback (+), the Luenberger observer (o) (left) and the proposed mismatched observer (*) (right)

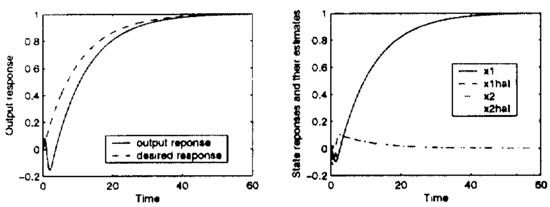

Figure 5: (a) Step response of the output y (left) (b) Step responses of internal modes x_1 and x_2 and their estimates $\hat{x}_1$ and $\hat{x}_2$ (right)

internal mode responses (x_1 and x_2) to a unit step input for the designed mismatched observer. The break line in (a) shows the ideal response (Equation (10)). Notice that this plant is a reverse reaction plant. Yet, the optimized observer feedback system approximates the ideal response well although it cannot remove the reverse reaction completely.

4.2 Application to Steering Control of Lateral Motion HDVs

4.2.1 Model Description

Under a couple of mild simplifying assumptions, the linearized model of lateral motion of a single-unit HDV (tractor-semitrailer type) can be given as follows [8].

$$\ddot{q} + A_{22}\dot{q} + A_{21}q = B_1\delta + B_2\dot{\epsilon}_d + B_3\ddot{\epsilon}_d \qquad (14)$$

where $A_{21} = M^{-1}K$, $A_{22} = M^{-1}D$, $B_1 = M^{-1}F$, $B_2 = M^{-1}E_2$, $B_3 = M^{-1}E_2$. $q = [y_r \; \epsilon_r \; \epsilon_f]^T$ is the generalized coordinate vector: y_r is the lateral displacement of the tractor's center of gravity, ϵ_r is the yaw angle of the tractor relative to the road, ϵ_f is the articulation angle between the tractor and semi-tractor. δ is the steering angle, and it is the control input. $\dot{\epsilon}_d$ and $\ddot{\epsilon}_d$ are the yaw rate and yaw acceleration of the road frame, respectively. They are regarded as disturbances. See [8] for detailed descriptions of the inertial matrix M, the damping matrix D, the stiffness matrix K, coefficient matrices F, E_1 and E_2. Note that M, D, F and K are functions of v (longitudinal velocity of the vehicle), m_2 (cargo loads in the trailer), and μ (road adhesion coefficient). Variations of these parameters are the main causes of model uncertainties. The input signal to the controller is defined as:

$$y_s = y_r + d_s\epsilon_r =: C_1 q \qquad (15)$$

where $C_1 = [1 \; d_s \; 0]$ and d_s is a constant called the "look-ahead" distance.

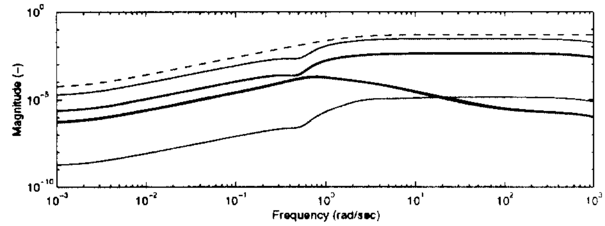

Figure 6: Frequency responses from disturbance input $\dot{\epsilon}_d$ to estimation errors $e(t) := x(t) - \hat{x}(t)$ (solid lines) and the inverse of the performance filter $W_o(s)$ (dashed line)

4.2.2 Controller and Observer Design

The control objective is to keep the lateral tracking error at the tractor center of gravity and the off-tracking error at the rear of the trailer to be small for lane-following maneuver. The objective of applying the feedback linearization controller to lateral control of the HDV model is to cancel all terms that are dependent on the longitudinal velocity, v, such that the desired disturbance-displacements dynamics can be obtained for any velocity.
First note that only A_{22} is dependent on v in (14). The feedback linearization controller (3) is designed for this plant. The controller gains are set to: $k_1 = 4.3294$, $k_2 = 3.2391$ and $k_3 = 1.7443$. These values are designed by using the H_∞ optimization algorithm presented in [7] such that the overall closed-loop system has the desired robustness against model uncertainties. The robustness of the entire closed-loop system is more likely determined by the choice of the controller gains of the feedback linearization controller, not the choice of the state observer. The state observer is designed to meet the internal stability requirement and the H_∞ performance requirement (7) with the following performance filter:

$$W_o(s) = \frac{s+2}{0.05(s+0.002)} \qquad (16)$$

The designed observer $C_o(s)$ is eighth order and achieves the closed-loop H_∞ gain of 1.162. Note that the order of the plant is sixth. The standard H_∞ control synthesis algorithm gives the controller of the same order as the combined system of the plant, the controller, and the performance filter. Figure 6 shows frequency responses of state estimation error dynamics, $F_L(P_K(s), C_O(s))$. The dashed line represents the frequency response of the inverse of $W_o(s)$.

4.3.3 Simulation Results

Time-domain simulations are conducted to show the closed-loop performance of the designed feedback linearization controller and H_∞ observer. The road curvature scenario for the simulations is designed based on the test track at Crows Landing Test Site [8]. Figure 7 shows the simulation results for the nominal condition and two perturbed conditions. Figure 8 shows the estimation error for each state variable. The maneuver is accomplished with an overshoot from the lane centerline less than 40 cm in all the cases. It can be observed that the designed feedback linearization controller gives more stable responses in a wider range of longitudinal veloci-

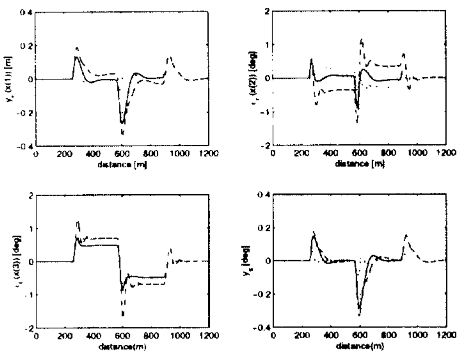

Figure 7: Closed-loop simulations: nominal condition (solid:v = 18 m/s, μ = 0.8, m_2 = 10670 kg) and perturbed conditions (dashed:v = 25 m/s, μ = 1.0, m_2 = 24000 kg) (dotted:v = 5 m/s, μ = 0.6, m_2 = 5000 kg)

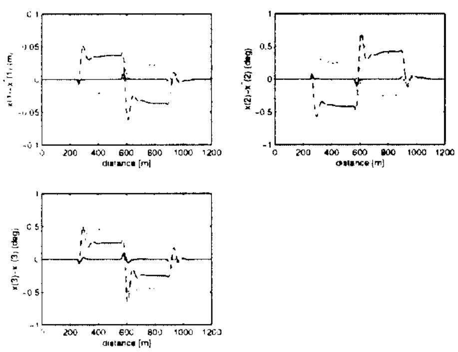

Figure 8: State estimation errors corresponding to (a)

ties than the time-invariant H_∞ controllers presented in [8]. Although the robustness is not the main scope of this paper, the designed controller and observer gives quite smooth responses even when parameter perturbations on m_2 and μ are introduced.

Figure 9 and 10 show the comparison between the direct state feedback case (i.e. all state variables of the plant are assumed available) and the case where the designed mismatched observer is used. Figure 9 shows the state responses under the perturbed condition and Figure 10 shows the closed-loop pole locations of each system. It can be seen that the closed-loop system with direct state feedback has the mode governed by the poles almost on the imaginary axis, which makes the output responses intolerably oscillating. On the other hand, the mismatched observer replaces those undamped modes by the modes governed by the poles near $(-0.1 \pm 0.5j)$ and thus it shows much more stable responses.

5 Conclusion

The major problem of the feedback linearization control scheme is the stability of the internal dynamics, which arises when the relative degree of the plant is less than its order. This paper presents the design methodology of a state observer that not only provides the controller with good estimation of the plant states, but also stabilizes the entire closed-loop system. By applying the proposed mismatched observer, the internal dynamics becomes observable from the output and affects the input-output dynamics since the separation principle no longer applies. By replacing the unstable internal dynamics by significantly faster modes, however, the effect of the new poles on the closed-loop input-output dynamics can be minimized. Two application examples were presented in this paper. In the second example, the proposed approach is applied to the steering control problem for lateral motion of heavy-duty vehicles. It was verified by numerical simulations that the proposed controller and observer structure showed favorable responses in a wide range of longitudinal velocities of the vehicle.

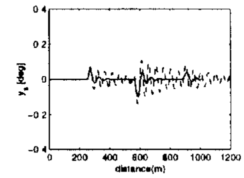

Figure 9: The closed-loop responses of $y_s(t)$ with direct state feedback(dashed) and the designed mismatched observer (solid) under the perturbed condition (v = 5 m/s, μ = 0.6, m_2 = 5000 kg)

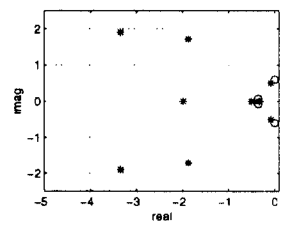

Figure 10: Closed-loop pole locations of the system with direct state feedback (o) and the designed mismatched observer (*) under the same conditions as in (a)

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